Stochastic Deficiencies Using Precise Point Positioning

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Abstract:

Precise Point Positioning (PPP) can be used for a wide range of applications in geodetic positioning. Providing precise clock and orbit information, a single station can be positioned in a reference frame. This leads to a very efficient processing procedure since the normal equation system is very small compared to network processing. PPP achieves very robust solutions comparable to network solutions.
A drawback of PPP is that it does not respect the principle of adjacent points, because correlations between neighbouring stations are not considered. This drawback leads to a disadvantageous stochastic behaviour between adjacent stations. However, as the point positions are computed using identical orbit and clock information, correlations between the estimated coordinates still remain. The presentation focus on this fact and determines correlations between station coordinates.

1. Introduction:

Generally geodetic positioning using the Global Positioning System (GPS) is based on the processing of simultaneously observed GPS data from several stations. Coordinate estimation with highest precision is performed using relative positioning. Forming single and later double differences between satellites and receivers eliminates the common errors contained in the GPS phase and code observations. (Algorithms based on undifferenced observations work similar and eliminate the common errors by parameter estimation). The clocks of the receivers and satellites account for the largest portion of the common errors. Other errors like e.g. ionosphere, troposphere and orbits are smaller, but their contribution to the error budget is still significant. While clock errors are completely removed by applying the double difference technique, the before mentioned errors will not completely removed by this technique since they are decorrelated with increasing distances between the receivers. For example, the atmospheric refraction as well as the orbit errors are different at two location, which are quite far from each other. In the case of the tropospheric refraction suitable models (Hopfield (1969), Saastomoinen (1973)) have to be applied for the estimation of the zenith delay in combination with adequate mapping functions (e.g. Niell (1996)) before forming the double difference observations. The double difference observations still contain a residual part of the tropospheric refraction, which is almost negligible for very short baselines but grows with distance between the stations. In case of the ionosphere it is recommended to use the ionospheric free linear combination, which is formed by dual frequency phase observations and absorbs the ionospheric refraction almost completely. It is possible to advert the orbit errors by using very precise orbit products of the International GPS Service (IGS), which are better than 5 cm.
In order to evaluate a large network of GPS stations it is necessary to process a large amount of data and to estimate a large number of parameters. This leads to rather large normal equation systems, which consume even on modern computers a lot of processing time.

2. Precise Point Positioning

As described above the computational effort increases with the size of GPS networks using relative positioning. Satellite clock errors have either to be estimated using the parameter estimation technique (undifferenced observation data) or eliminated using the double difference technique. In both cases it is necessary to process the data of all stations used in a network, which have tracked the GPS satellites simultaneously. Therefore the processing time increases with the size of the network. As an alternative approach *Precise Point Positioning* (PPP) has been developed, which supplies the user with precise orbit and clock information. The Precise Point Positioning (Zumberge et. al. 1997)) has originally been developed at the Jet Propulsion Laboratory (JPL). PPP is now also a processing option realised in the new BERNESE 5.0 (Hugentobler et. al. 2005). This paper will only focus on the PPP strategy as it is applied with GIPSY/OASIS II developed at JPL.
A globally distributed network of GPS receivers is used to estimate precise GPS satellite positions and satellite clock corrections. Using this information simplifies the burden for the estimation of the remaining parameters. The satellite clock and orbit information can be downloaded from a data server at JPL. This information can then be used to analyse the data of a single station and estimate the station specific parameters like the coordinates, receiver clock and the tropospheric refraction. The ionospheric refraction is accounted for by using the ionospheric free linear combination of the carrier phase observations. Therefore this technique is only applicable for dual frequency receivers and sufficient observation times supposing 1 cm accuracy. PPP allows therefore only the estimation of the site specific parameters. This reduces the size of the linear equation system significantly and herewith also the processing time. The processing time for a network of GPS stations using PPP is linearly dependent on the number of stations. Standard relative positioning using a least squares algorithm leads to normal equation systems that grow with the square root of the number of stations. Even though the square root information filter (SRIF) (Bierman 1977) used by GIPSY/OASIS II is still slower by a factor of 2 than ordinary least squares algorithm, PPP decreases the processing time of GPS network

significantly. It is also an advantage of PPP that it allows easy diagnosis of receiver specific problems. In case of the double differencing technique always three stations have to be analysed to be certain about a receiver specific problem at one site. Is the problem identified in PPP only one single station needs to be reprocessed and not the complete network consisting of a large number of stations. Sophisticated scientific software is of course in most cases capable of identifying outliers, while it does data screening. Hence, a reprocessing can be avoided in many cases.
The satellite clock corrections are either sampled at 300 or 30 (high rate) seconds, while the satellite positions and velocities are given only every 15 minutes as the ordinary IGS orbits. Receiver (site) specific parameters are estimated using these two products. The coordinates of a GPS network can then be determined by a simple adjustment of all coordinate components. Of course the clock corrections and satellite positions have to be defined in a certain reference system realisation. JPL offers two possibility to process the GPS data: either the data are processed in a non-fiducial frame (Heflin et. al. 1992) or the products are available in a specific reference frame. The coordinates estimated with the non-fiducial products have to be transformed by a simple 7-parameter transformation. Both coordinate results are consistent. The advantage of the non-fiducial approach is that reference frame changes can simply compensated by using an appropriate set of transformation parameters. Reprocessing is unnecessary.
The effectiveness and the reliability of PPP has been proven in many cases. The coordinates of the SIRGAS (Sistema de Referencia Geocéntrico para America del Sur) network have been processed by three analysis centres, which were the DGFI, BEK and IBGE. While DGFI (Deutsches Geodätisches Forschungsinstitut) and IBGE (Instituto Brasileiro de Geografia e Estatística) were using BERNESE, the BEK (Bayerische Kommision für die Internationale Erdmessung) has been using GIPSY/OASIS II applying PPP and computed a very consistent set of coordinates (Drewes et. al. 2005). Nevertheless one drawback of PPP remains: ambiguity fixing is not possible using the data of just one site together with the satellite orbits and clock corrections. Therefore the horizontal components in west-east-direction are poorer estimated compared to fixed solutions.
While relative positioning uses the data of several receivers tracking the same satellites, the GPS observation data are naturally correlated. They contain the same clock errors of the satellites as well as the effects of the orbit and atmospheric errors. The estimated absolute coordinates are well correlated as it can be seen from their variance-covariance matrix. Of course this is not the case for PPP. The results of the individual stations combined in an adjustment would give no correlation information between the absolute coordinates from different sites. This cannot be quite true since the observation data are correlated by the identical satellite clock errors, orbit errors and finally by the atmospheric refraction. The aim of this paper is to investigate the remaining correlation between GPS sites, which is neglected by the PPP strategy.

3. The impact of the Correlation coefficients

GIPSY/OASIS II gives as a result of a PPP estimation three cartesian coordinate components of a single station with its covariance matrix Σ_{XX} shown in equation 1.

$$\Sigma_{x_i x_i} = \begin{bmatrix} \sigma^2_{x_i} & \rho_{xy}\sigma_{x_i}\sigma_{y_i} & \rho_{xz}\sigma_{x_i}\sigma_{Z_i} \\ \rho_{xy}\sigma_{x_i}\sigma_{y_i} & \sigma^2_{y_i} & \rho_{xz}\sigma_{x_i}\sigma_{z_i} \\ \rho_{xz}\sigma_{x_i}\sigma_{y_i} & \rho_{yz}\sigma_{x_i}\sigma_{z_i} & \sigma^2_{z_i} \end{bmatrix} \quad (1)$$

The covariance matrix of a GPS network Σ_{XX} estimated with the PPP strategy is a banded matrix composed of 3 by 3 element submatrices on the diagonal, which contain the error estimates of the individual station i. All other elements of the matrix are zero. The design of the covariance matrix is shown in equation 2.

$$\Sigma_{XX} = \begin{bmatrix} \Sigma_{x_1 x_1} & 0 & \cdots & 0 \\ 0 & \Sigma_{x_2 x_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma_{x_n x_n} \end{bmatrix} \quad (2)$$

Following the principle of relative positioning it is well known that the individual error components are quite large. Therefore the absolute point error of a GPS position can be in the order of several metres depending on the duration of the observations. The absolute point error of a station cannot be improved using relative positioning, but taking the correlations of the observations into account the relative coordinate components can be estimated with an accuracy of a few millimetres. The correlations of the observations are reflected in the covariance matrix of the estimated absolute coordinates by the correlation coefficients between the different parameters.
Equation 3 describes the law of error propagation. The covariance matrix of the relative coordinates $\Sigma_{\Delta X \Delta X}$ is a function of the coefficient matrix A, here for only two stations, and covariance matrix Σ_{XX} for the absolute coordinates.

$$\Sigma_{\Delta X \Delta X} = A \cdot \Sigma_{XX} \cdot A^T \quad , with \quad A = [I \quad -I] \quad (3)$$

The coefficient matrix A is composed of two unity matrices I, where the second unity matrix has a negative sign. For example, the error of the relative coordinate component Δx_{12} can be computed applying the law of error propagation as written in equation 4:

$$s^2_{\Delta x_{12}} = s^2_{x_1} - 2 \cdot \rho_{x_1 x_2} \cdot s_{x_1} \cdot s_{x_2} + s^2_{x_2} \quad (4)$$

The equation shows quite clearly that the estimates become more accurate than the absolute point positions as soon as the correlations are larger than 0.5. This is generally the case for relative positioning. In case of PPP the estimates of the individual coordinate components are treated uncorrelated, due to the processing strategy. Therefore the error of a relative coordinate component grows simply by the square root of 2 compared to the absolute coordinate components (see equation 4, for ρ=0 and $s_{x1} = s_{x2}$).

Figure 1 shows the time series of coordinate residuals for the stations REYK and REYZ established in Reykjavik on Iceland. Both stations are located within two metres from each other. While the station REYK tracks only GPS data, the receiver on station REYZ tracks GPS and also GLONASS data. But for both stations the coordinates were estimated using only GPS data. Similarities between the two time series can be seen with a glance on the figure. Especially the residuals of the north component are obviously correlated, which is an indication that the estimated coordinates of adjacent points are still correlated.

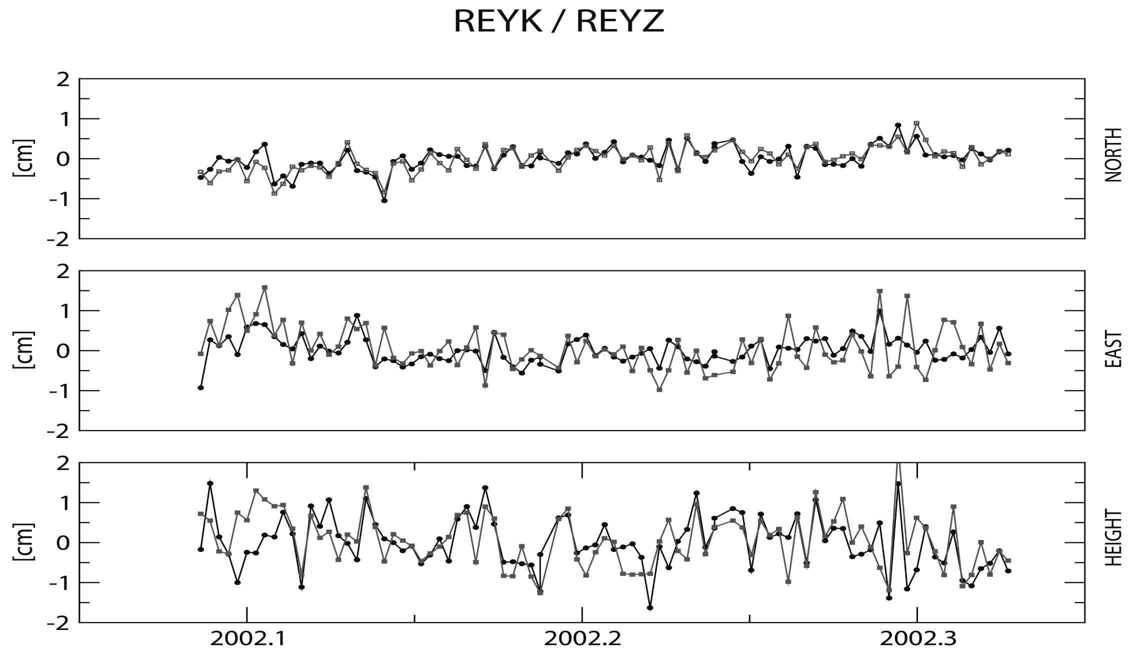

Figure 1: Residuals of coordinate estimates from the two sites REYK and REYZ on Iceland in north, east and height given in centimetres.

It is quite obvious that there are still remaining correlations between the estimated coordinates, which can be visualized in the time series but cannot be treated by the GIPSY/OASIS II analysis. The reason for this remaining correlations are influences from the common environment and the impact of common errors like orbits, clocks and other remaining error contributions. The stations REYK and REYZ are collocated GNSS sites, where the impact is greatly expected. It is therefore interesting to extent the analysis to stations with larger separation.

4. Analysis

The analysis of the correlations is based on coordinate estimates using GIPSY/OASIS II with the PPP strategy. The empirical correlation coefficient *r* of two data samples *u* and *v* can be determined using equation 5.

$$r_{u,v} = \frac{\sum_{i=1}^{k} (u_i - \bar{u})(v_i - \bar{v})}{\sqrt{\sum_{i=1}^{k} (u_i - \bar{u})^2 \cdot \sum_{i=1}^{k} (v_i - \bar{v})^2}} \quad (5)$$

This equation can be applied for the determination of the correlation between any coordinate component and any two stations. Hence, the full correlation information can be derived empirically between all coordinate components.

A network consisting of seven station located in central Europe was selected (compare figure 3). It consists of the stations ONSA, WTZR, LROC, ZIMM, GRAS, MATE and NOT1. The network covers the central part of Europe and has baseline lengths between 350 and 2300 km. There are several conditions for the empirical estimation of the correlation between two sets of coordinates that had to be fulfilled:

- the time series have to cover the same period of time
- the coordinates have to be estimated under the same conditions
- trends have to be removed
- outliers need to be eliminated

A time period had to be defined for the PPP coordinate estimation that meets these pre-requisites. During the time window changes in the set-up of the station were not allowed. Therefore an arbitrary selection of the time window was not possible. In April 2003 the receiver and antenna was replaced in Caussols (GRAS) and in August 2003 the equipment was exchanged in Zimmerwald (ZIMM) and Onsala (ONSA). Finally a period of approximately 100 days was chosen between May and August 2003, where no equipment changes took place.

The procedure for the estimation of the coordinates was also of importance. Therefore the Niell mapping function was used for the tropospheric parameter estimation on each site. Clock corrections given every 300 seconds and orbit parameters were applied. Clock corrections every 30 seconds were also available but were not utilised. Ocean loading corrections were applied to remove any possible signal that could have an effect on the correlation coefficient. The GPS data were processed for each day under these conditions and coordinates derived in the ITRF2000.

One should also consider that stations located on the same continental plate are subject to almost the same plate motion. The signal induced by the plate motion over 100 days is still small but will increase the correlation between these two pairs of coordinates. If the stations are located on two different continental plates, the plate motion of the two different plates will decrease the correlation. Hence, the estimates for the relative accuracy between adjacent points will be falsified by the effect of the plate motion. It is therefore essential that a linear trend is removed from the time series of the coordinate estimates. The removal of the linear trend compensates for the plate motion, since plate motions are generally treated as linear velocities.

In a last step outliers in the time series had to be detected and removed. At the same time gaps were filled by linear interpolation. Gaps occur as well for outliers, since the existing data were removed, as for missing GPS data. After this step has been carried out, the procedure for the estimation of the correlation coefficient could proceed.
Equation 5 can be used to estimate the correlation coefficients between all coordinate components empirically. The correlation matrix $\rho_{X_i X_j}$ between two points i and j can be written as

$$\rho_{X_i X_j}=\begin{bmatrix} \rho_{x_i x_j} & \rho_{x_i y_j} & \rho_{x_i z_j} \\ \rho_{y_i x_j} & \rho_{y_i y_j} & \rho_{y_i z_j} \\ \rho_{z_i x_j} & \rho_{z_i y_j} & \rho_{z_i z_j} \end{bmatrix} \quad (6)$$

For each component in the matrix equation 5 is applied to estimate the empirical correlation coefficient between two sets of data. The complete correlation matrix of a network consisting of n points is given as:

$$R_{XX}=\begin{bmatrix} \rho_{X_1 X_1} & \rho_{X_1 X_2} & \cdots & \rho_{X_1 X_n} \\ \rho_{X_2 X_1} & \rho_{X_2 X_2} & \cdots & \rho_{X_2 X_n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{X_n X_1} & \rho_{X_n X_2} & \cdots & \rho_{X_n X_n} \end{bmatrix} \quad (7)$$

Together with the standard deviation of each component (e.g. the standard deviation of the component x of station n) the empirical covariance matrix $\hat{\Sigma}_{XX}$ for the whole network can be computed and used for further analysis.

$$\hat{\Sigma}_{XX}=diag\left[\sigma_1 \quad \sigma_2 \quad \ldots \quad \sigma_n\right]\cdot R_{XX}\cdot diag\left[\sigma_1 \quad \sigma_2 \quad \ldots \quad \sigma_n\right] \quad (8)$$

5. Results

The empirical covariance matrix $\hat{\Sigma}_{XX}$ has been computed analysing the time series of the coordinates for the 7 stations shown in figure 3. The standard deviation of each coordinate component has been estimated from its repeatability and has not been taken from the PPP analysis. Generally the standard deviations derived from GPS processing software are too optimistic. Standard deviations derived from repeatability give more realistic numbers. It is known from relative positioning that the correlation between two stations is dependent on the distance between the two stations. This is quite obvious since two sites, like those in Reykjavik, underlie the same tropospheric refraction.

Therefore the coordinate estimates should be highly correlated. This is also valid for the other error components. One can expect that a similar behaviour can be seen from this analysis. Figure 2 shows therefore the correlations between the coordinate components in a cartesian and a local horizontal system between the 7 stations in dependence of the baseline length.

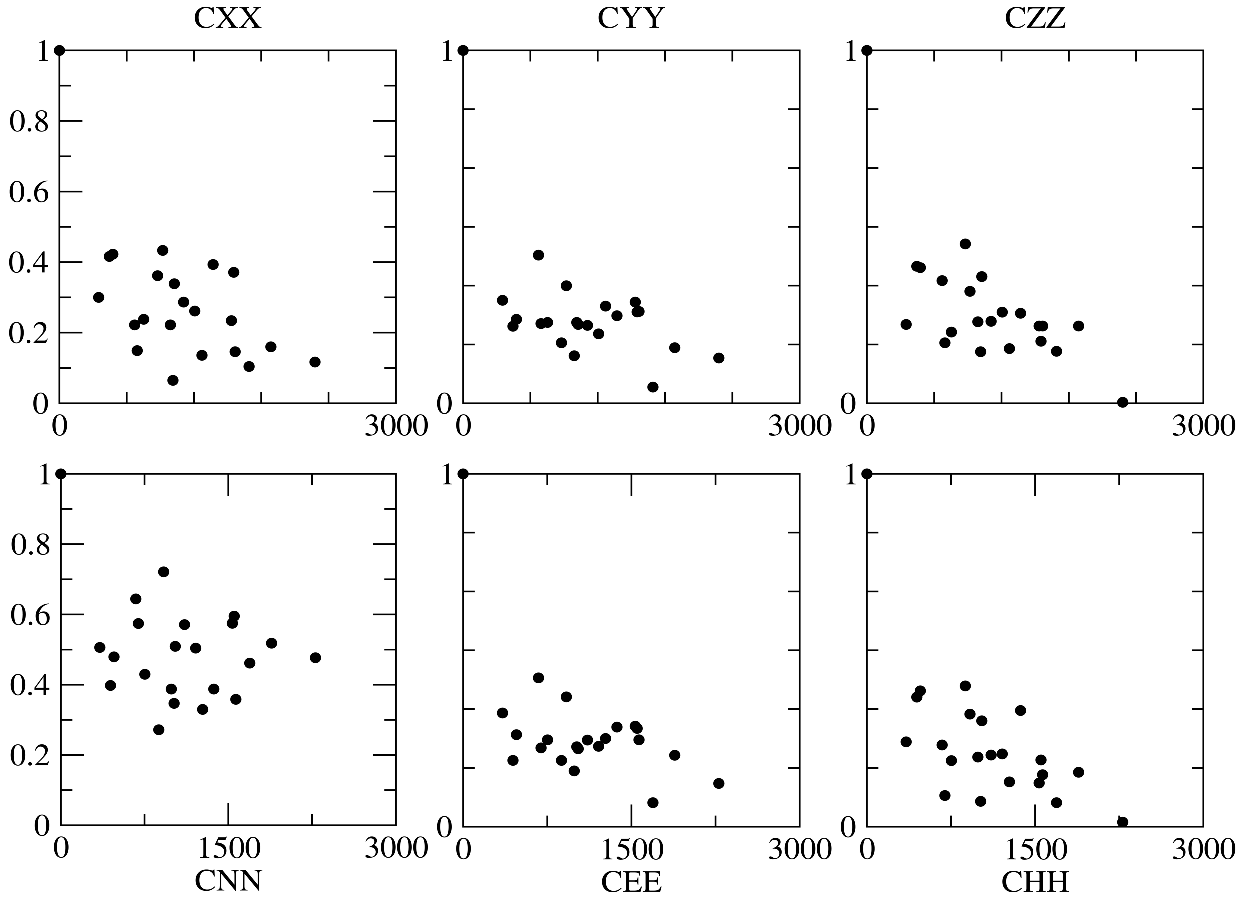

Figure 2: Correlation coefficients in cartesian and local ellipsoidal components in dependence of the baseline length.

From a first glance at the graphs in figure 2 a dependency between baseline length and correlation coefficient cannot clearly been seen. Only in the region between 1500 and 2300 km it appears, that the correlation decreases with longer baseline lengths. But for shorter baseline lengths the scattering is quite large and therefore the derivation of the correlation coefficient as a well as defined function of the baseline length have no solid basis. While the cartesian correlation coefficient have a very similar distribution and seem to scatter around almost the same mean value (0.29 for X, 0.26 for Y and 0.28 for Z), the scatter for horizontal components is different. Here the largest correlation coefficient can be seen for the north component. The mean of the correlation coefficient is about 0.50 for the north component, while it is 0.27 for the east and 0.24 for the height component.

SITES		Length [km]	**Without Correlation**			**With Correlation**		
			[mm]					
			North	East	Height	North	East	Height
ONSA	LROC	1535	2,8	4,7	6,4	1,8	4,0	6,0
ONSA	WTZR	920	2,8	5,0	6,8	1,5	4,0	5,6
ONSA	ZIMM	1207	2,9	4,9	6,7	2,0	4,3	6,0
LROC	ZIMM	670	2,9	5,0	7,2	1,7	3,8	6,3
WTZR	ZIMM	476	2,9	5,3	7,7	2,1	4,6	6,0
GRAS	WTZR	753	3,0	5,9	7,7	2,2	5,1	7,0
GRAS	ZIMM	350	3,1	5,8	7,8	2,2	4,8	6,8
GRAS	MATE	877	3,6	6,6	9,2	3,1	5,9	7,3
MATE	WTZR	1013	3,5	6,1	9,2	2,9	5,4	8,9
LROC	NOT1	1690	2,8	5,7	8,0	2,1	5,5	7,8
ONSA	NOT1	2280	2,9	5,6	8,0	2,1	5,3	7,9

Table 1:Relative position accuracy derived from precise point positioning using selected baselines from the 7-station network.

The results of the empirically estimated correlation coefficient is reflected in table 1. It shows selected baselines of the

7-station network with their baseline length and the relative position accuracy. The standard deviations are computed once for ρ=0 applying equation 4 on the left side and once with empirically estimated correlation coefficient r on the right side of the table. Comparing both sides of the tables shows that the standard deviation of the relative positions decreases applying this strategy. Therefore it is proven that the precise points positioning strategy contains a larger potential of accuracy than it is stated from the processing results. Especially the north component is improved. In general one can see an improvement of roughly 30 % over the standard results for the north component. At the same time the relative accuracy for the east and height component are improved only slightly. The east component shows an improvement of only 13% and the height component of only 10%. Two numbers which are not very impressive.

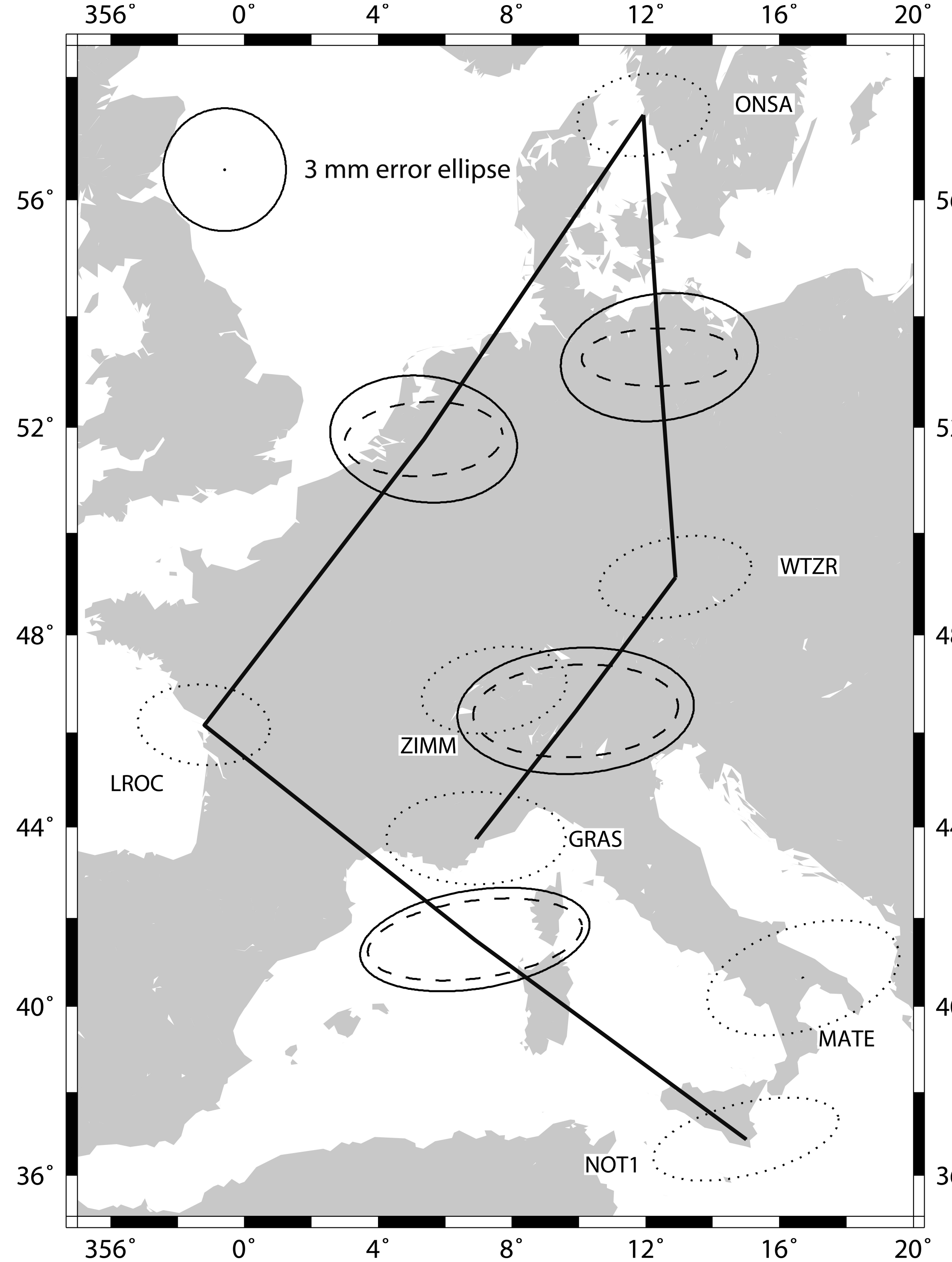

Figure 3: Test network for the estimation of the correlation coefficient between coordinate components. The maximum baseline length is about 2300 km. Horizontal error ellipses with dotted lines show absolute error components, solid lines show error ellipses using standard PPP and dashed lines show relative error ellipses computed from the empirically estimated correlations (using PPP).

The results are visualised in figure 3 for the horizontal components. The figure shows that the absolute point error ellipses derived with PPP are clearly more accurate than the relative error ellipses. Applying the empirical correlation estimation cannot overcome this effect. Nevertheless, an improvement over the standard relative error ellipses is visible. It can be clearly seen that the relative position accuracy has been improved in the north component for the baselines between Onsala (ONSA) and Wettzell (WETZ) and for the baseline between Onsala and La Rochelle (LROC). On the other hand this figure shows also some unfavourable examples with the baseline between Wettzell and Caussols (GRAS) and between La Rochelle and Noto (NOT1). The baseline between Noto and La Rochelle shows only marginal improvements for the horizontal component. The rather large distance (~ 1700 km) between these two sites may account for this. Generally the example also shows that this technique improves the relative position accuracy for the longitudinal component only very slightly. This effect is caused by the missing ambiguity estimation. The PPP strategy does not allow ambiguity estimation for a single station. It is well known that the so called "Float-Solutions" show relative error ellipses as they are formed in figure 3; the semi-major axis of the error ellipses is orientated along the longitudinal component and is by a factor of two larger as the semi-minor axis.

6. Conclusion:

The PPP strategy applied within GIPSY/OASIS II has shown its effectiveness and operability on large networks, even on global networks. Positions can be estimated at the level of a 5-10 mm applying only satellite clock and orbit information, which have been derived by JPL within a global network. Orbit and clock information provided by JPL can be used by any receiver without distinction of it geographical position. A drawback of this method has been stated already: the principle of adjacent points is neglected. Stations in the vicinity of each other are treated independently without considering errors stemming from common sources (e.g. atmosphere). Therefore, the coordinate results are treated as being not correlated, which of course is not the case.
This analysis of the coordinate time series of different sites has shown that the different coordinate components are still correlated. Significant correlation coefficients can be seen in the latitudinal component, while the correlations in the longitudinal and height component are only small. The rather small correlation in the longitudinal component can be addressed by the missing ambiguity estimation. Due to the unresolved ambiguities the times series are too noisy for the estimation of a clear correlation signal. This is also somehow the case for the height component. The height component still remains to be the weakest portion of the position estimates applying GNSS techniques.
One should expect that concepts applied in Real Time Kinematic GPS networks like the concept of the *Virtual Reference Station* (VRS) or the *FKP-concept* (FKP= *Flächenkorrekturparameter*) should address the problem of the principle of adjacent point in a much better way. In both cases the correction parameters are dependent on the location of the GPS site and derived from GPS data in the vicinity of the site. The disadvantage of these concept is of course that it does not work on a global scale as the JPL concept does.
This analyse has shown that it is possible to recover parts of the correlation information between neighbouring stations, which leads to an improvement of the accuracy of relative coordinate components. However, for the rigorous estimation of the correlations between the sites, in order to follow the principle of adjacent points, remains only the simultaneous processing of all observation data.

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