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Faculty of Land Engineering & Geodesy

Time and frequency analysis of daily reprocessed solutions from selected EPN stations

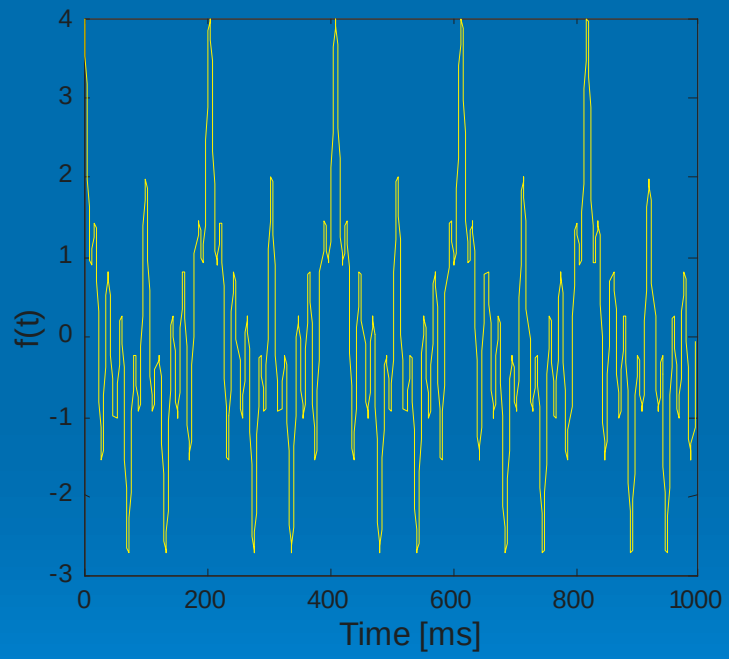
Authors:
Paweł Kamiński
Krzysztof Kroszczyński
Mariusz Figurski

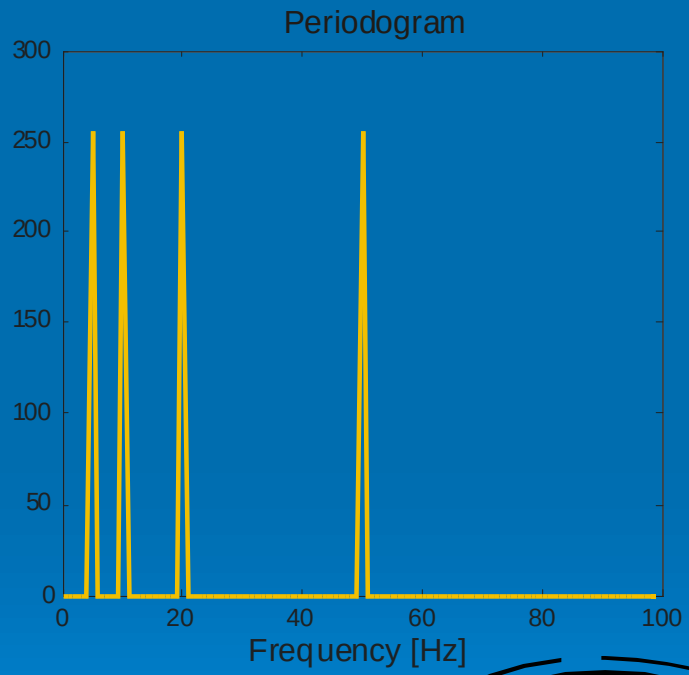
Centre of Applied Geomatics

Speech Arrangement

- Examples of frequency evaluation with respect to model examples (FFT and wavelet)
- Selection of 3 groups of EPN stations to evaluate time series of ‘h’ coordinates
- Presentation of model frequency analysis by means of two techniques
- Conclusions

Fourier transform is the most popular method of frequency detection in time signals





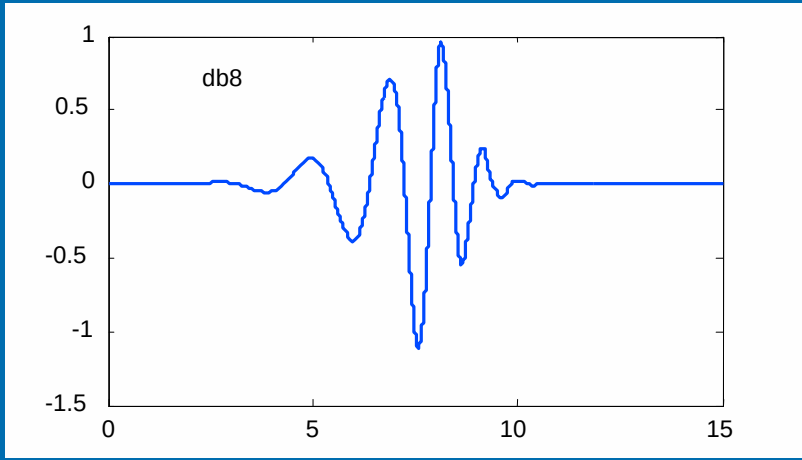
Coordinates analysis need different tools such as FFT (Fast Fourier Transforms) as well as wavelet transform.

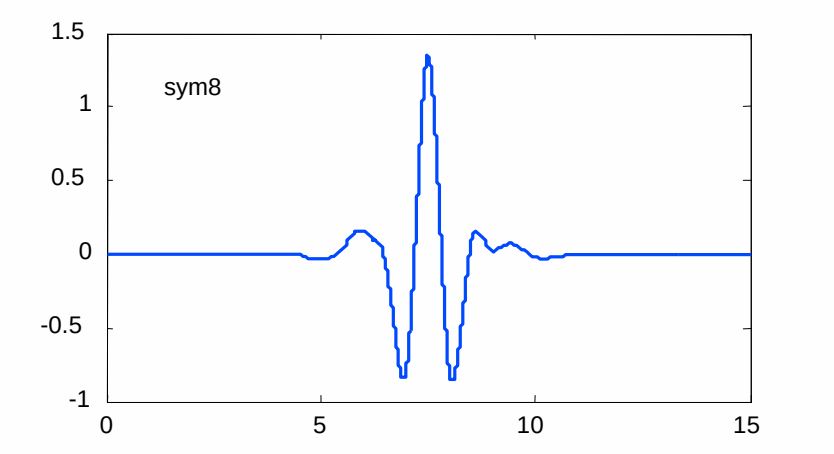
Wavelet transform is adopted in case of non-stationary signals, i.e. signals carrying components of frequency changing in time. To understand wavelet transform we need to use convolution of a given signal with the functions received by scaling and shifting of so called **mother wavelet**.

$$\gamma(s,\tau) = \int f(t)\psi^{*}_{s,\tau}(t)dt = \int \frac{1}{\sqrt{s}} f(t)\psi^{*}\left(\frac{t-\tau}{s}\right)dt$$

Coefficient of wavelet transform is the measure of corelation of investigated signals and analysed wavelet. 's' corresponds to the scale parameter (inversion of frequency), time lag mother wavelet described by parameter τ and $s=1$. Due to factor $\tau=0$ wavelet energy of $\sqrt{s}$ analysed wavelet is not changing with the scale.

Example of two mother wavelets





Mother wavelet is well concentrated as far as time and frequency. These conditions determine that this function has at least a few oscillations. That is why it is a ‘short wavelet’.

$$\int \psi(t)dt = 0 \qquad \Psi(\omega) = 0$$

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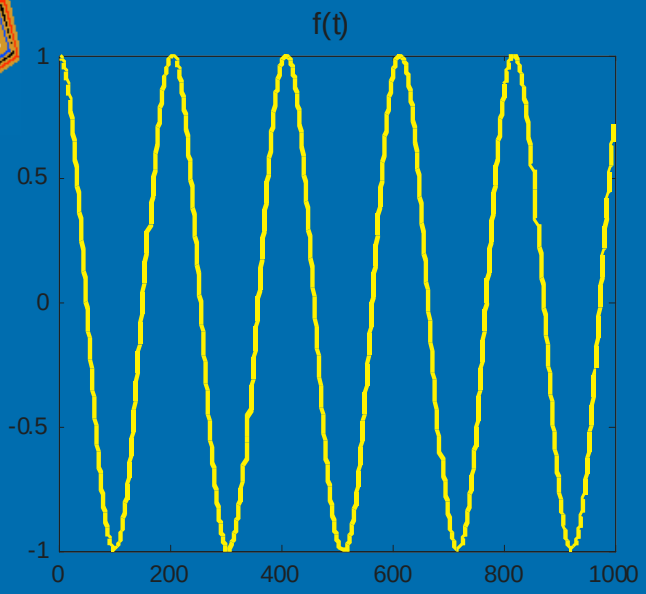

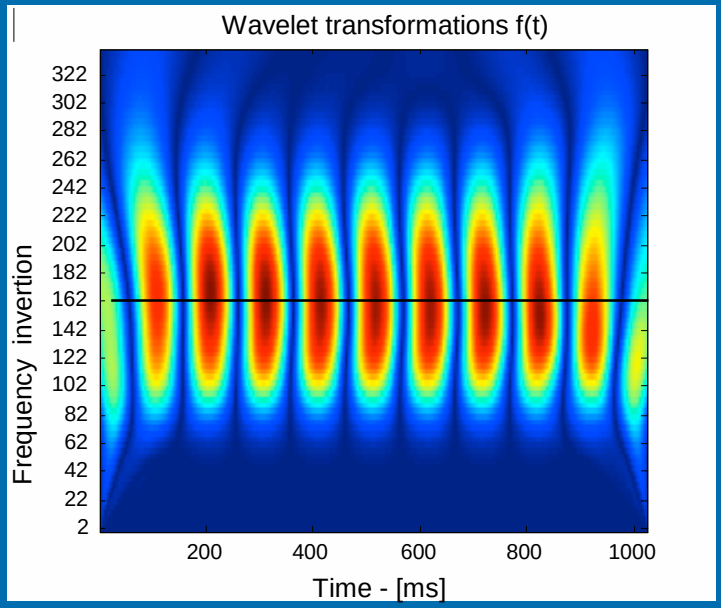

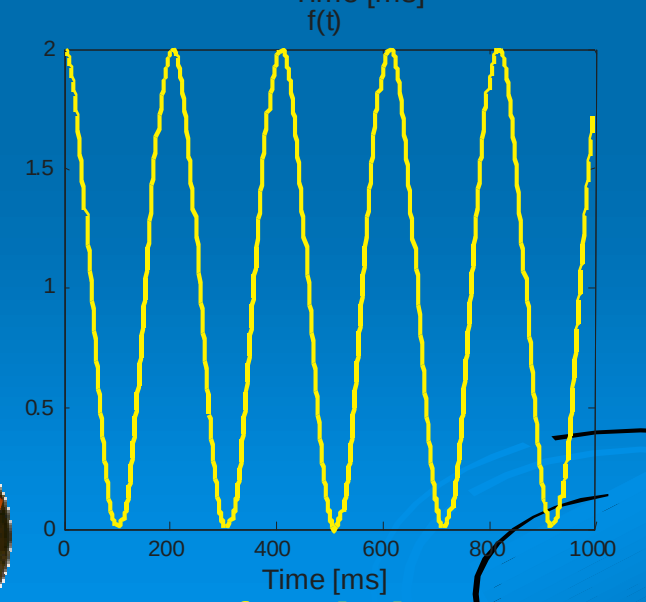

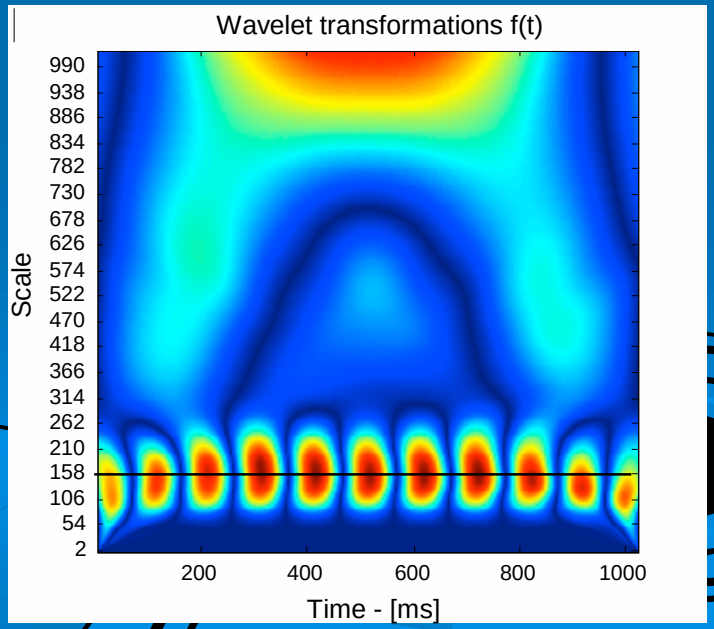

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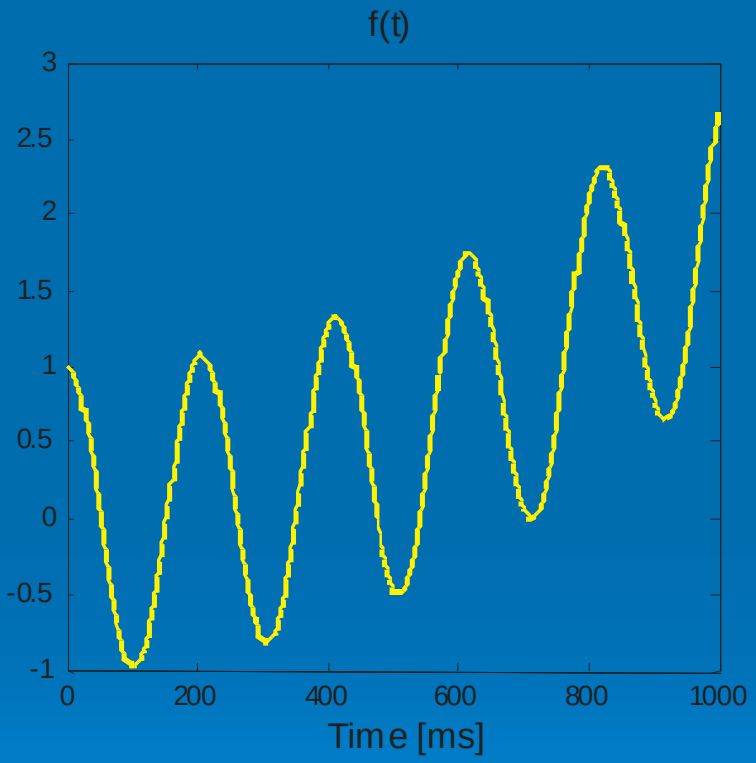

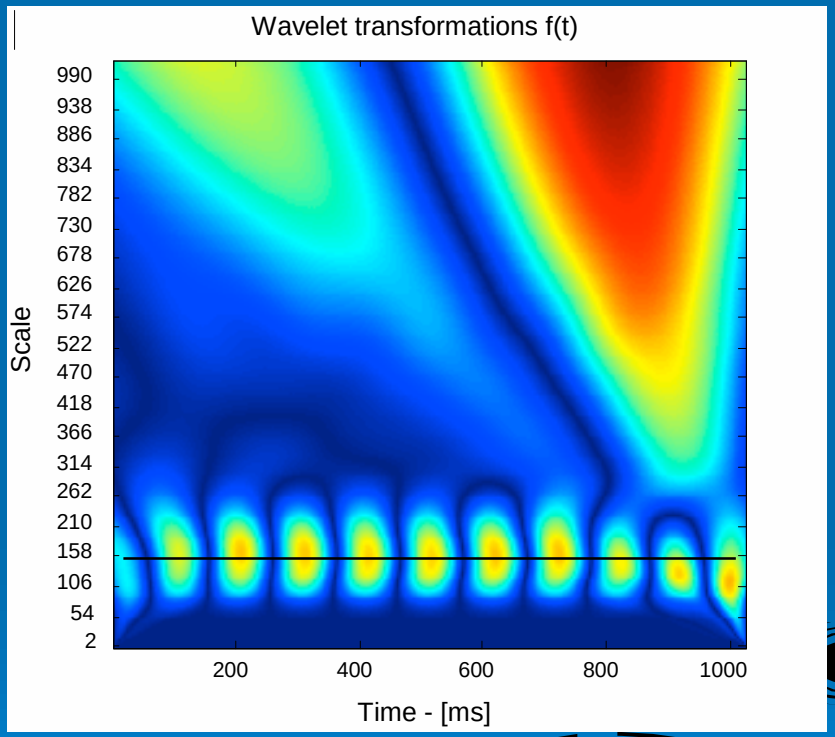

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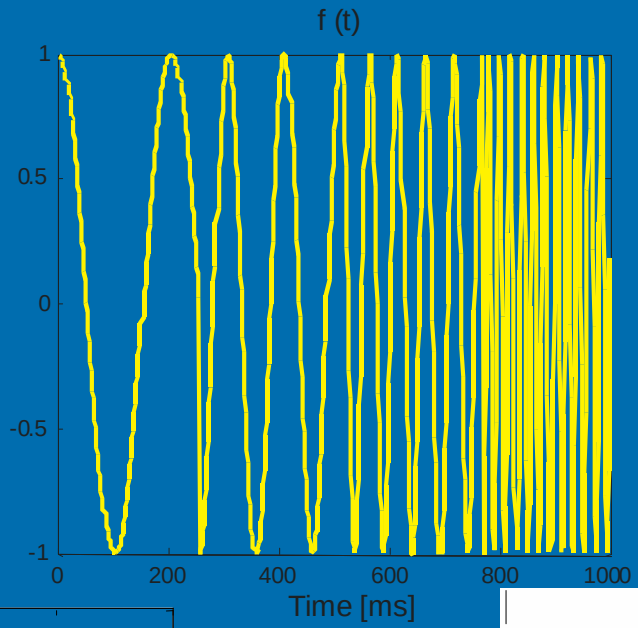

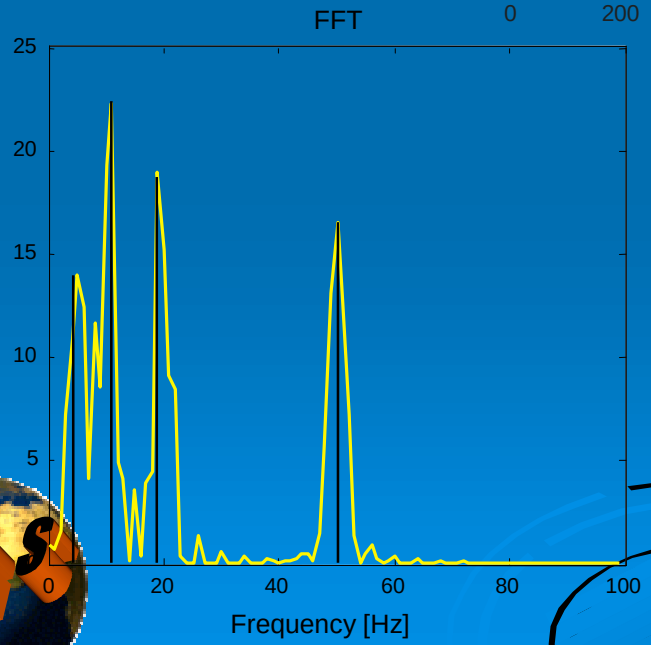

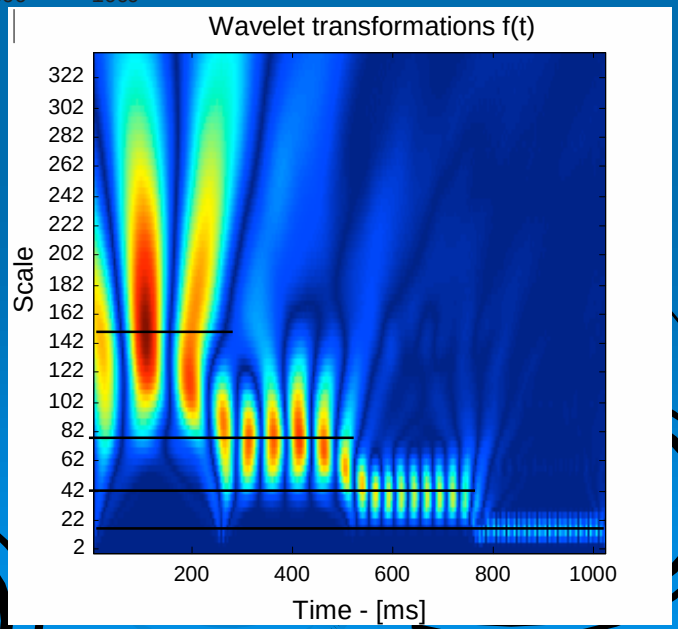

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LONG

ZIMM, VILL, POTS, ONSA, METS, MAS1, KOSG, KIRU, JOZE, GRAZ, GRAS

These stations have time series of 4654 - common days observations starting from 2.04.1995.

MIDDLE

VIL0, TRO1, RIGA, RAMO, NYA1, MARS, KIR0, JOEN, GLSV, GENO

These stations have time series of 3225 - common days observations starting from 1.03.1999.

SHORT

SULD, ZYWI, SKE0, DELF, EIJS, TERS, COMO, EVPA, MSEL, TUC2

These stations have time series of 1170 - common days observations starting from 15.10.2004.

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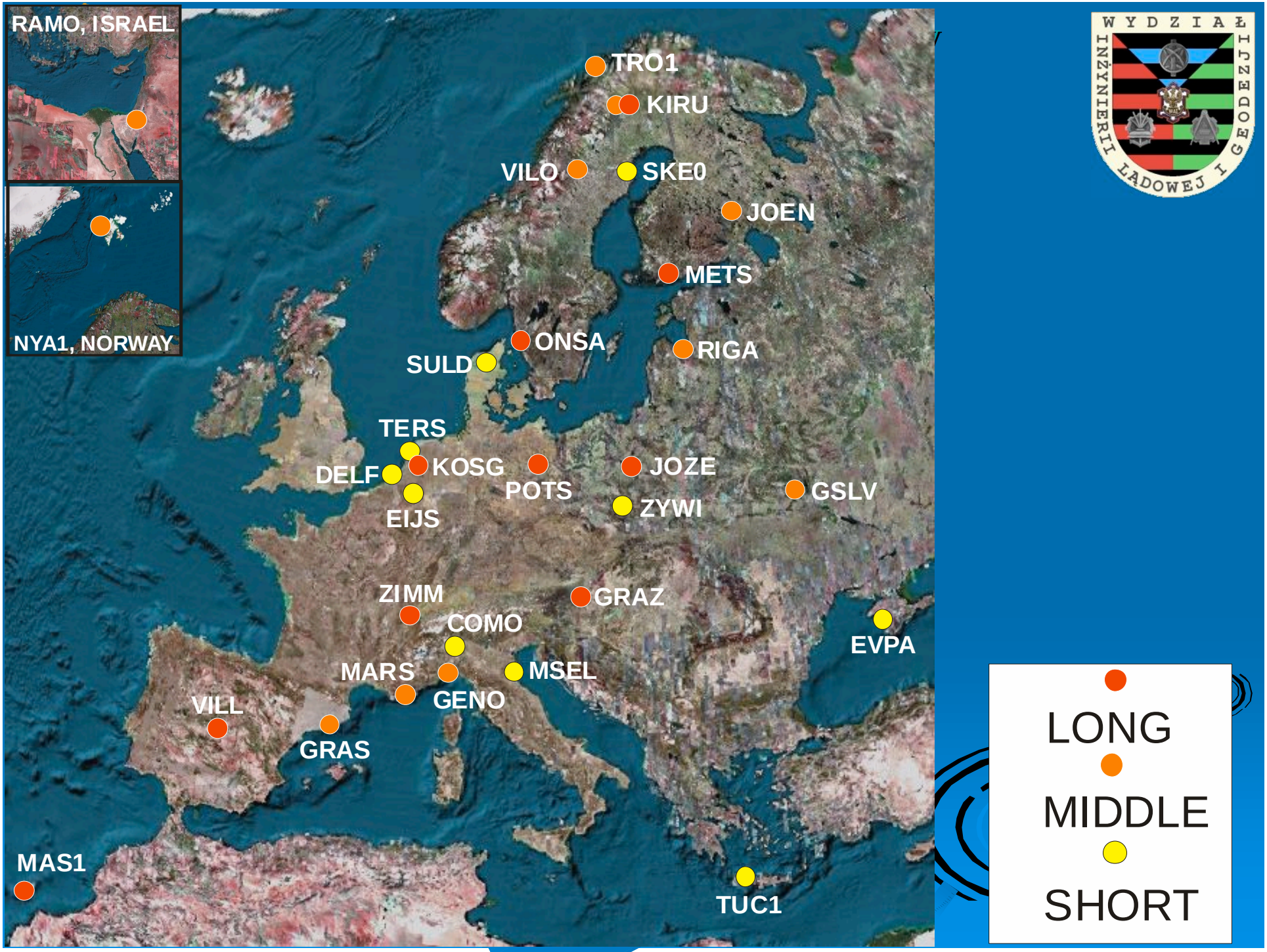
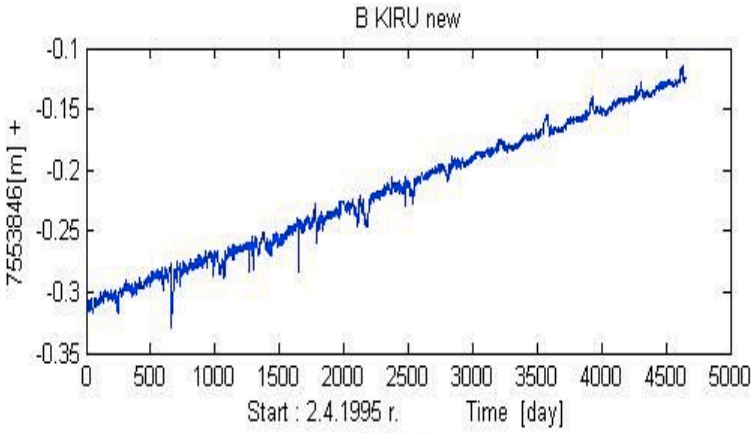
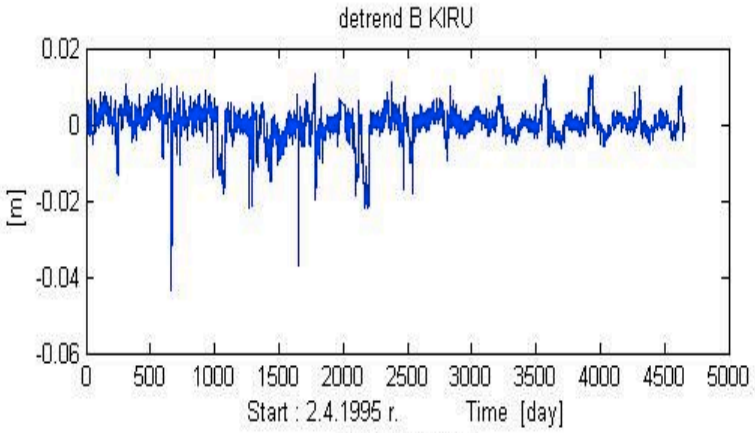
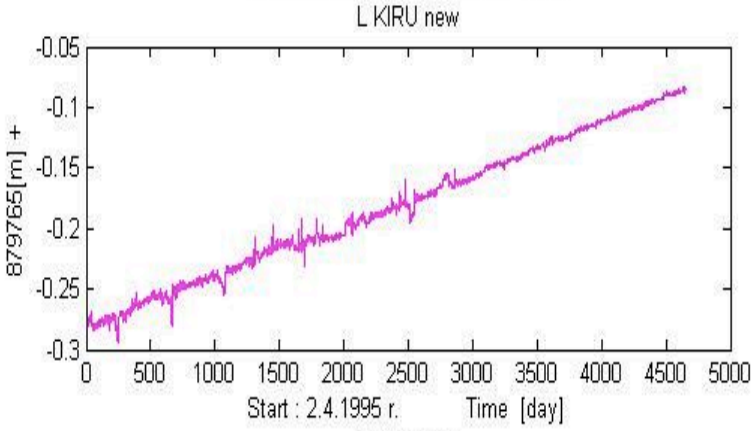
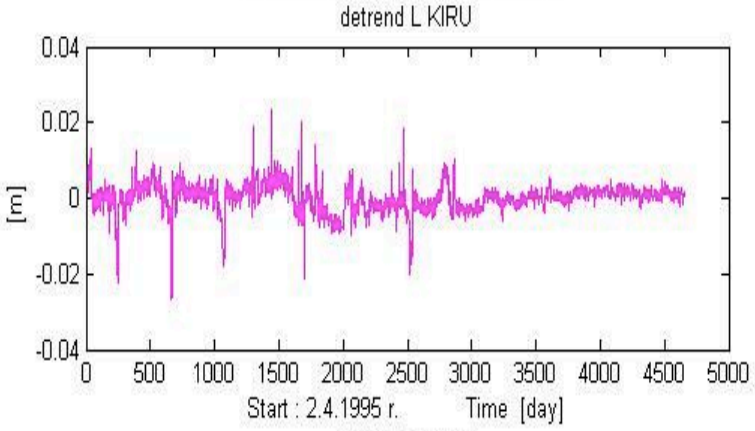
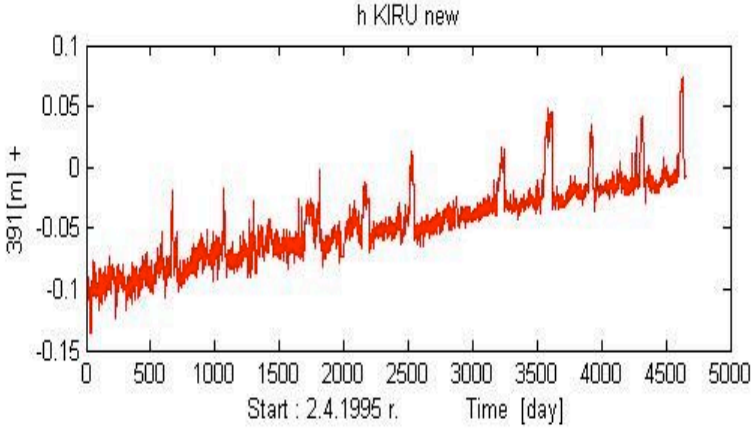

RAMO, ISRAEL
NYA1, NORWAY
TRO1
KIRU
VILO
SKE0
JOEN
METS
ONSA
RIGA
SULD
TERS
KOSG
DELF
POTS
JOZE
GSLV
EIJS
ZYWI
ZIMM
GRAZ
COMO
EVPA
MARS
MSEL
GENO
VILL
GRAS
MAS1
TUC1
LONG
MIDDLE
SHORT

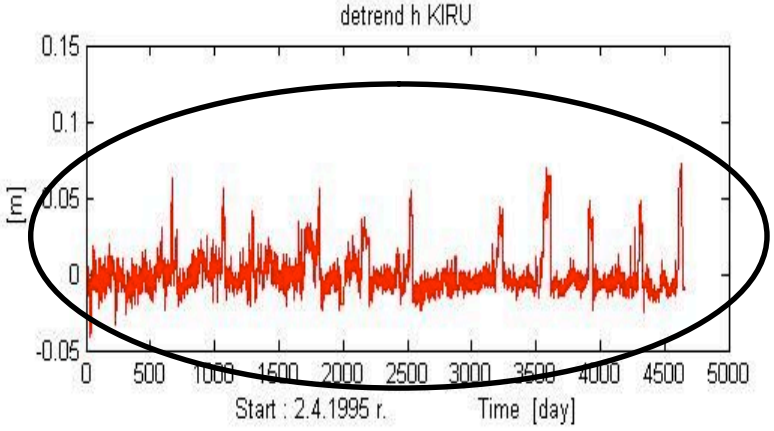

B KIRU new
7553846[m] +
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Start : 2.4.1995 r.
Time [day]

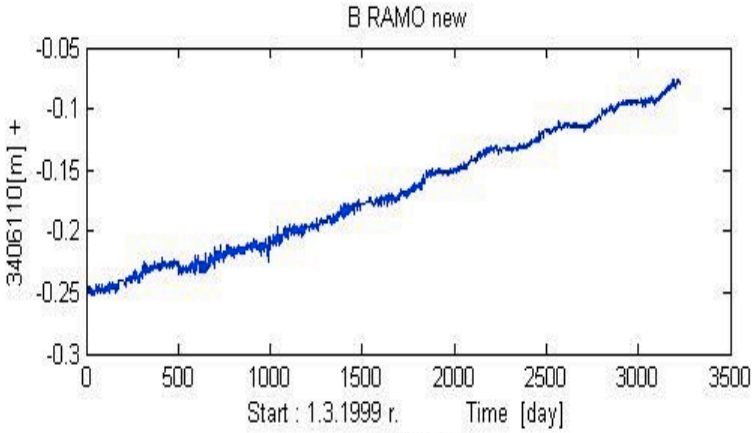

detrend B KIRU
[m]
0.02
0
-0.02
-0.04
-0.06
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Start : 2.4.1995 r.
Time [day]

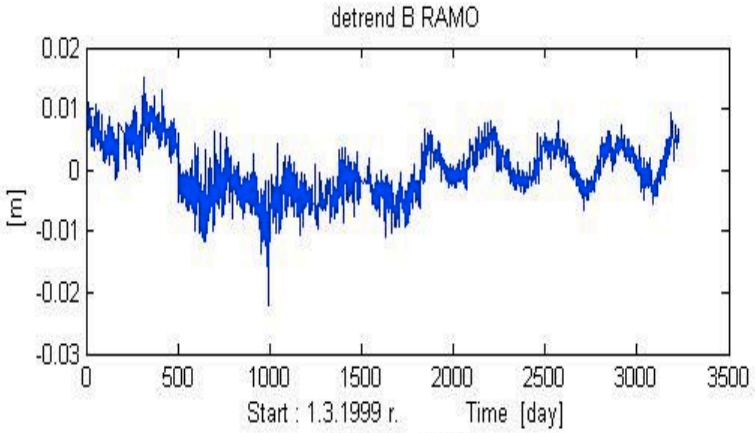

L KIRU new
879765[m] +
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Start : 2.4.1995 r.
Time [day]

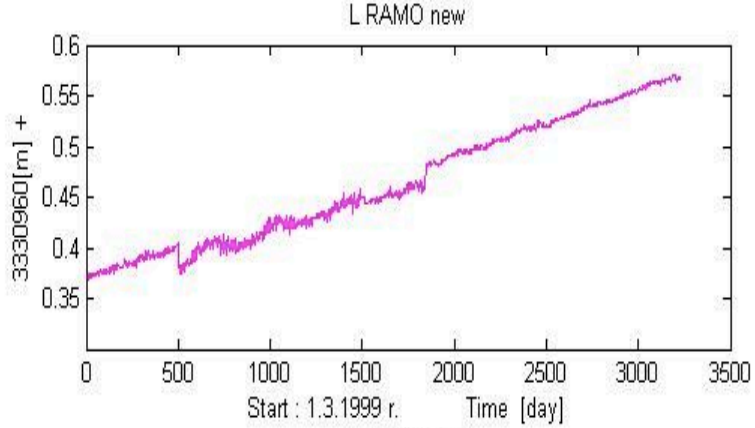

detrend L KIRU
[m]
0.04
0.02
0
-0.02
-0.04
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Start : 2.4.1995 r.
Time [day]

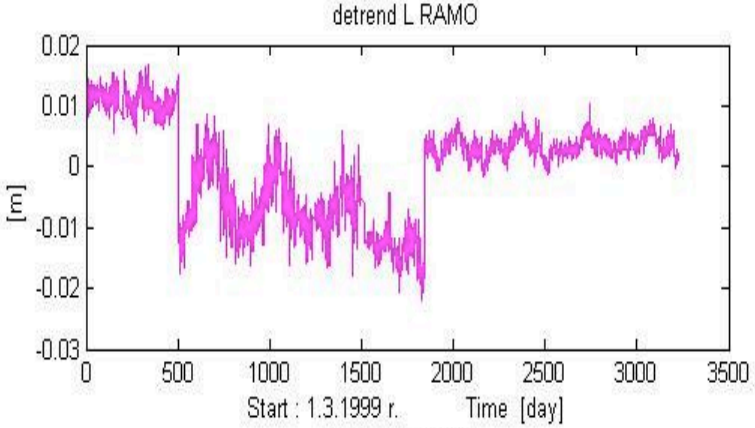

h KIRU new
391[m] +
0.1
0.05
0
-0.05
-0.1
-0.15
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Start : 2.4.1995 r.
Time [day]

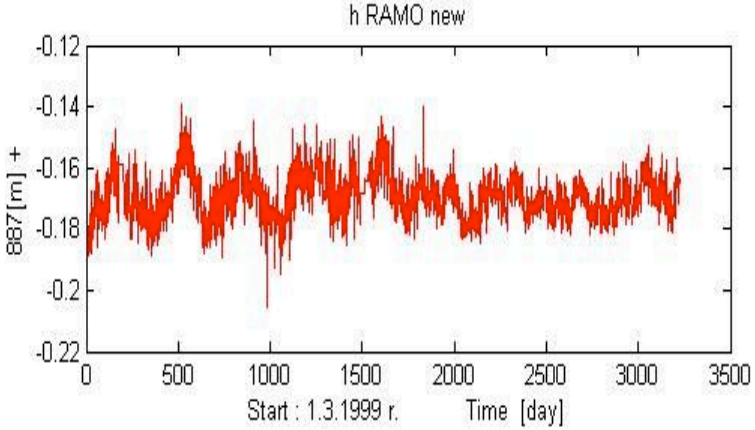

detrend h KIRU
[m]
0.15
0.1
0.05
0
-0.05
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Start : 2.4.1995 r.
Time [day]

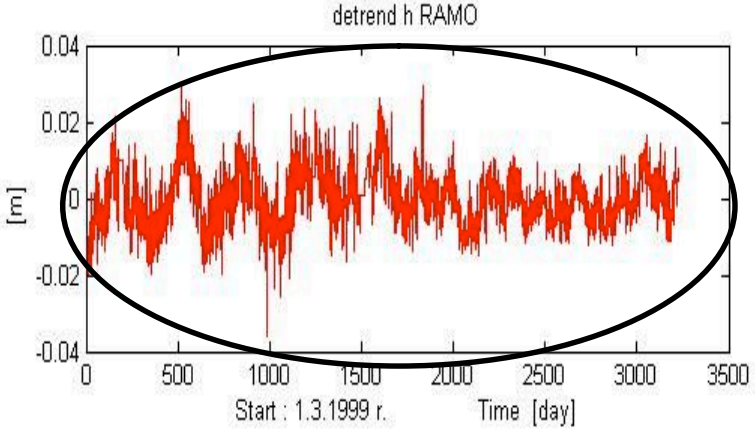

B RAMO new
3406110[m] +
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
0
500
1000
1500
2000
2500
3000
3500
Start : 1.3.1999 r.
Time [day]

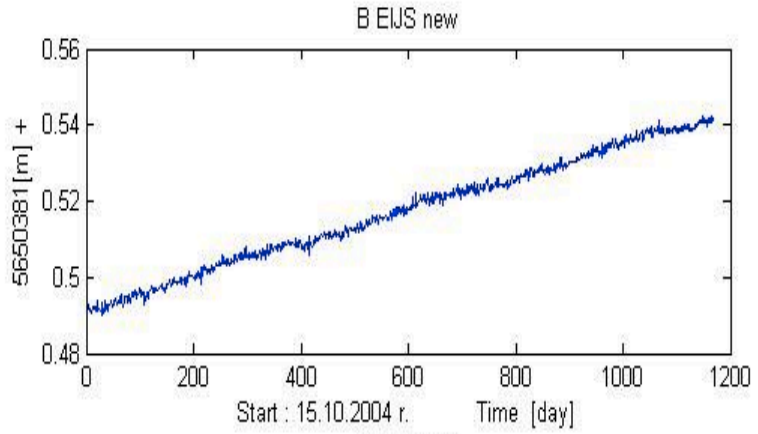

detrend B RAMO
[m]
0.02
0.01
0
-0.01
-0.02
-0.03
0
500
1000
1500
2000
2500
3000
3500
Start : 1.3.1999 r.
Time [day]

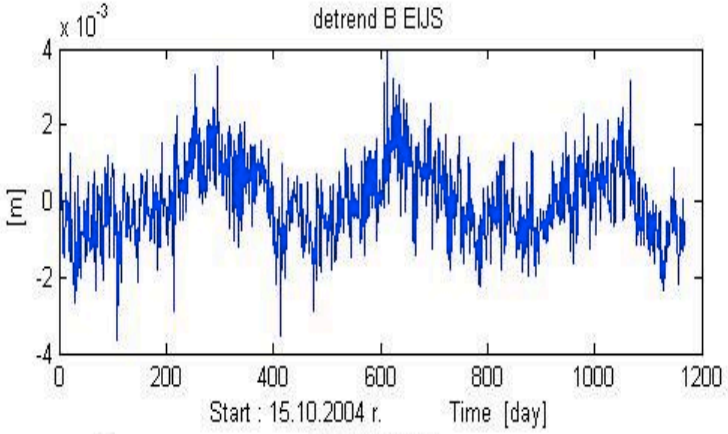

L RAMO new
3330960[m] +
0.6
0.55
0.5
0.45
0.4
0.35
0
500
1000
1500
2000
2500
3000
3500
Start : 1.3.1999 r.
Time [day]

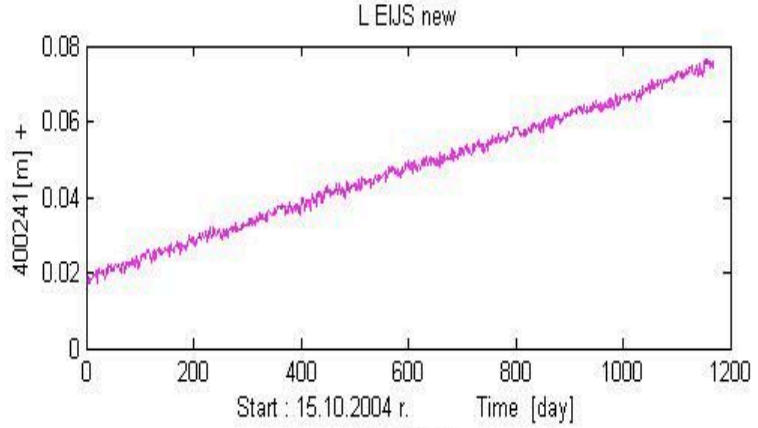

detrend L RAMO
[m]
0.02
0.01
0
-0.01
-0.02
-0.03
0
500
1000
1500
2000
2500
3000
3500
Start : 1.3.1999 r.
Time [day]

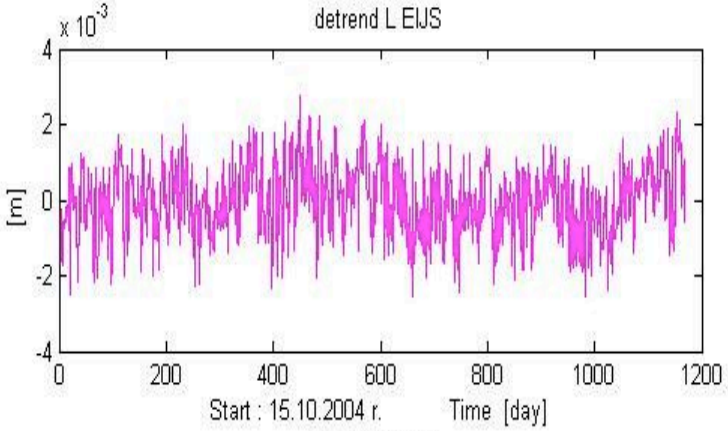

h RAMO new
887[m] +
-0.12
-0.14
-0.16
-0.18
-0.2
-0.22
0
500
1000
1500
2000
2500
3000
3500
Start : 1.3.1999 r.
Time [day]

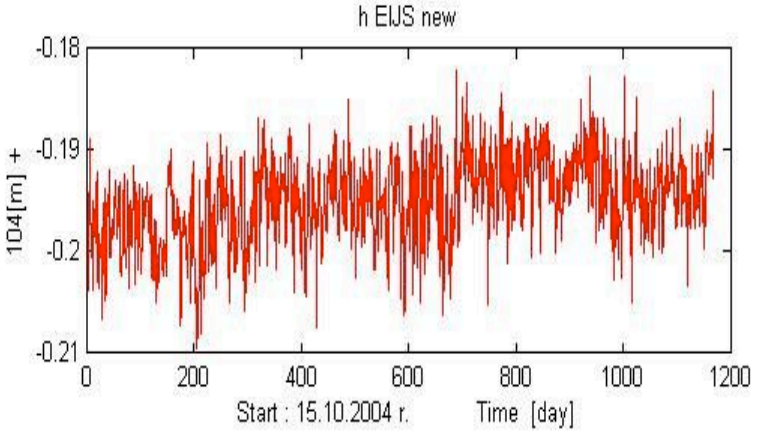

detrend h RAMO
[m]
0.04
0.02
0
-0.02
-0.04
0
500
1000
1500
2000
2500
3000
3500
Start : 1.3.1999 r.
Time [day]

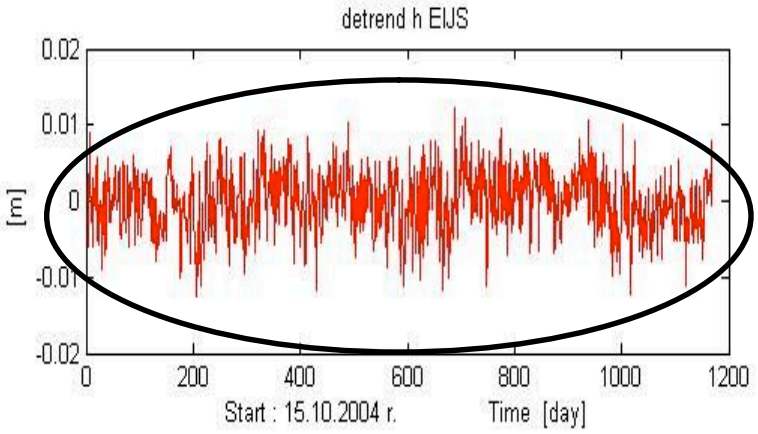

B EIJS new
5650381 [m] +
0.56
0.54
0.52
0.5
0.48
0
200
400
600
800
1000
1200
Start : 15.10.2004 r.
Time [day]

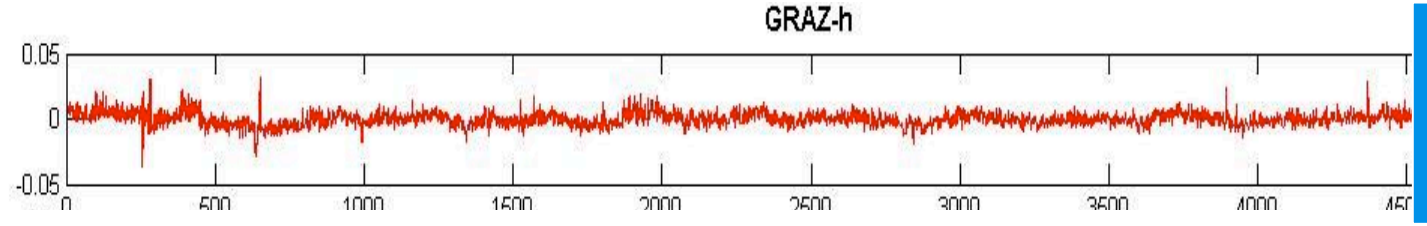

detrend B EIJS
x 10^-3
[m]
4
2
0
-2
-4
0
200
400
600
800
1000
1200
Start : 15.10.2004 r.
Time [day]


L EIJS new
400241 [m] +
0.08
0.06
0.04
0.02
0
0
200
400
600
800
1000
1200
Start : 15.10.2004 r.
Time [day]

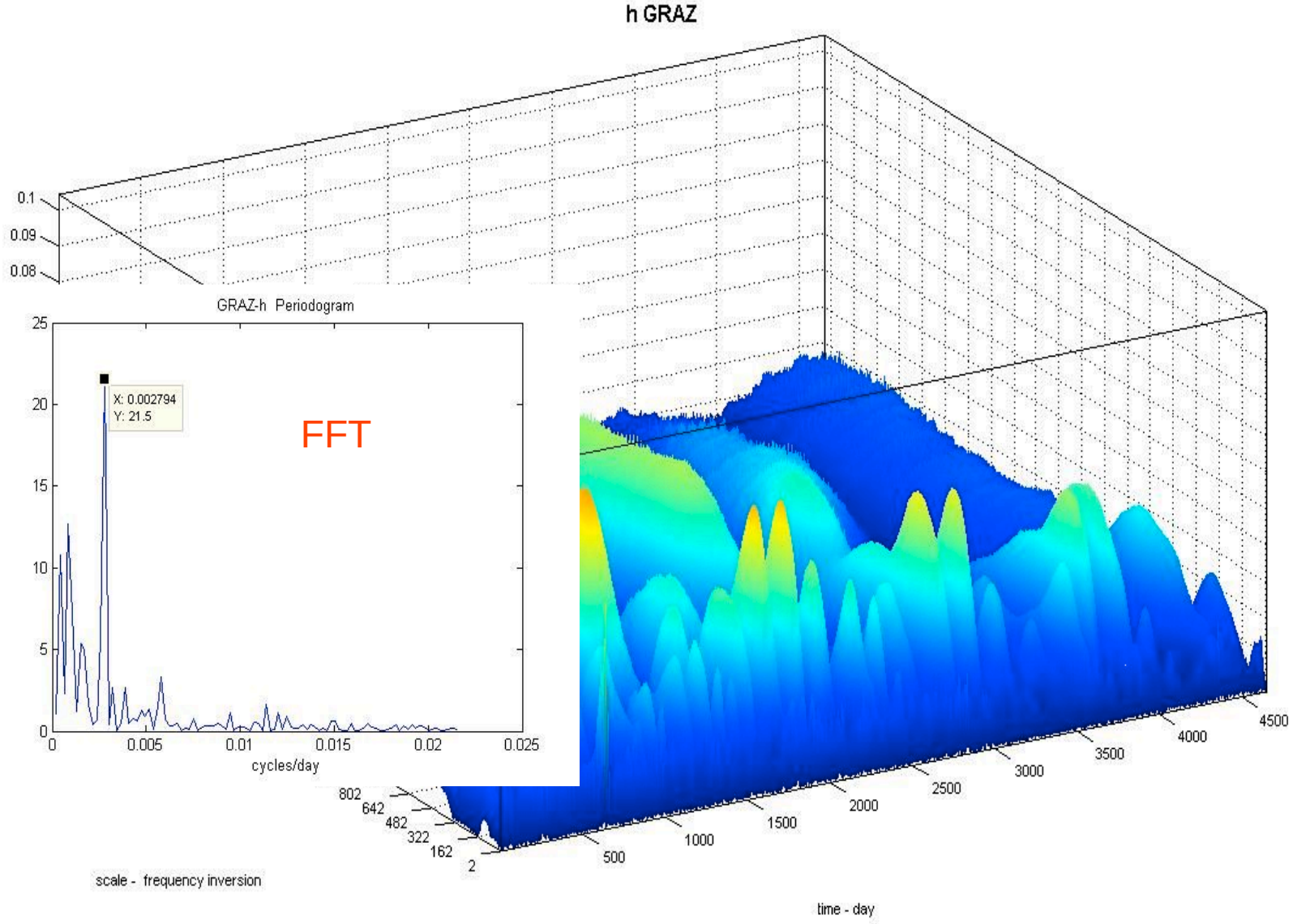

detrend L EIJS
x 10^-3
[m]
4
2
0
-2
-4
0
200
400
600
800
1000
1200
Start : 15.10.2004 r.
Time [day]

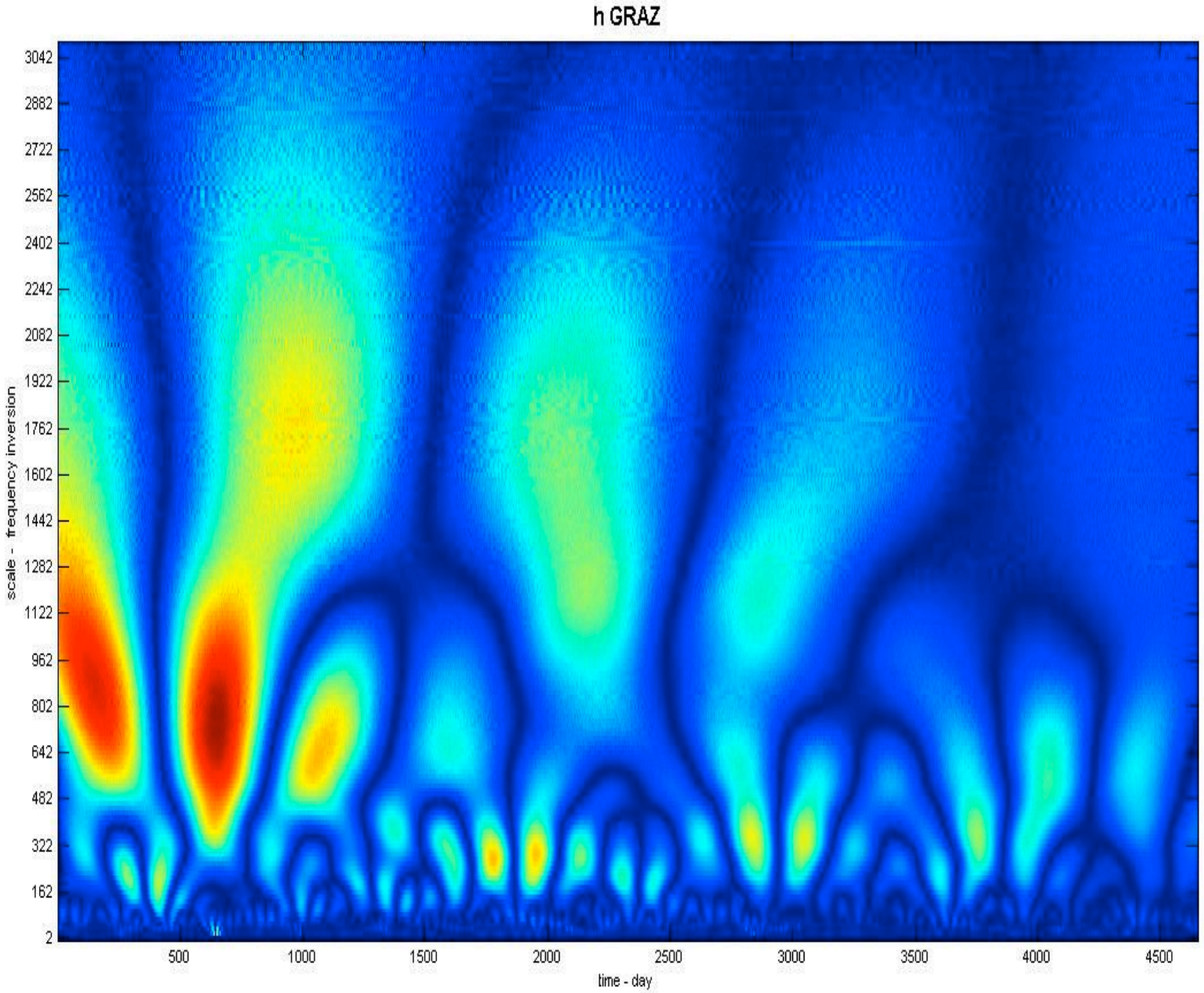

h EIJS new
104 [m] +
-0.18
-0.19
-0.2
-0.21
0
200
400
600
800
1000
1200
Start : 15.10.2004 r.
Time [day]

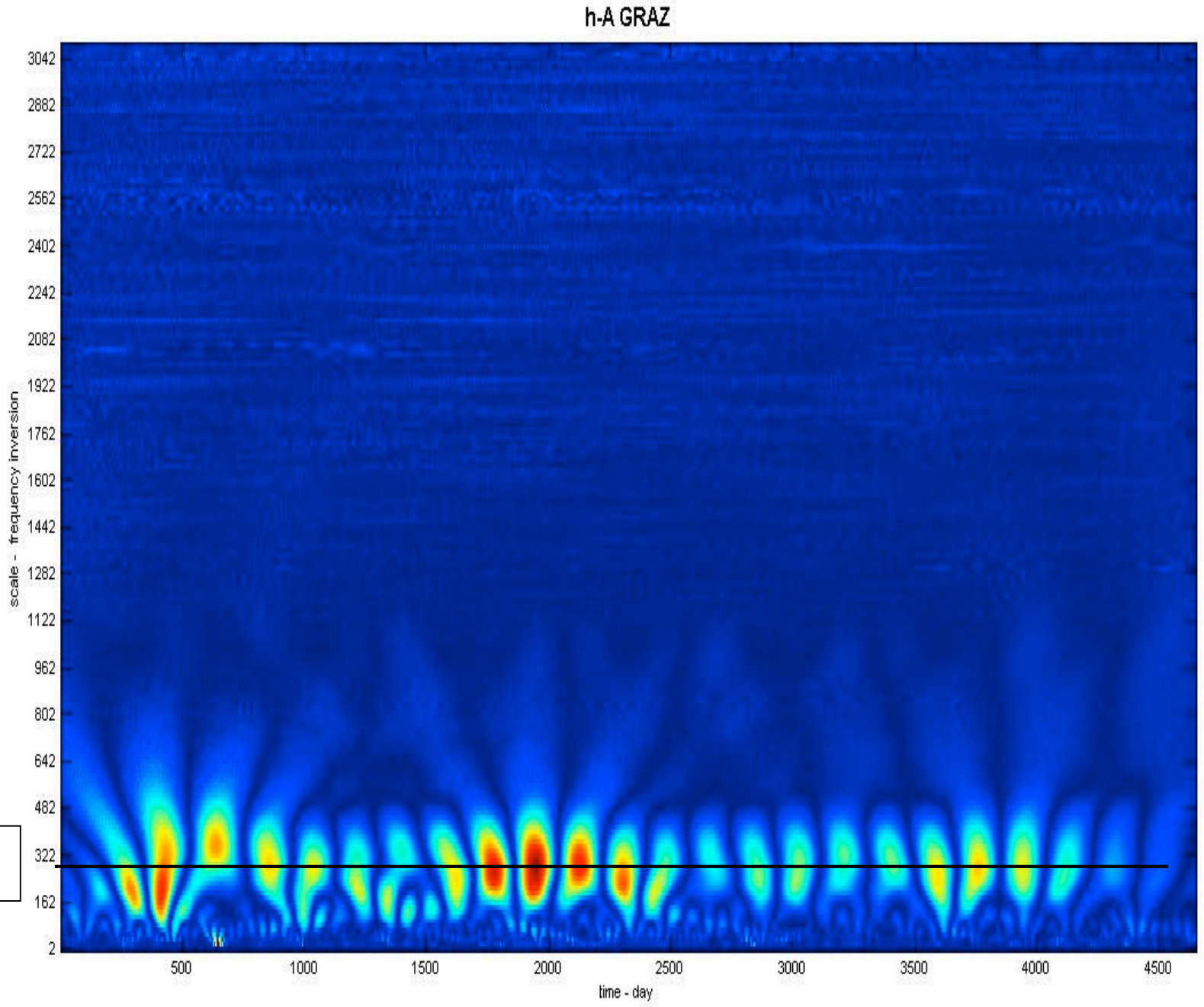

detrend h EIJS
[m]
0.02
0.01
0
-0.01
-0.02
0
200
400
600
800
1000
1200
Start : 15.10.2004 r.
Time [day]


GRAZ-h
0.05
0
-0.05
0
500
1000
1500
2000
2500
3000
3500
4000

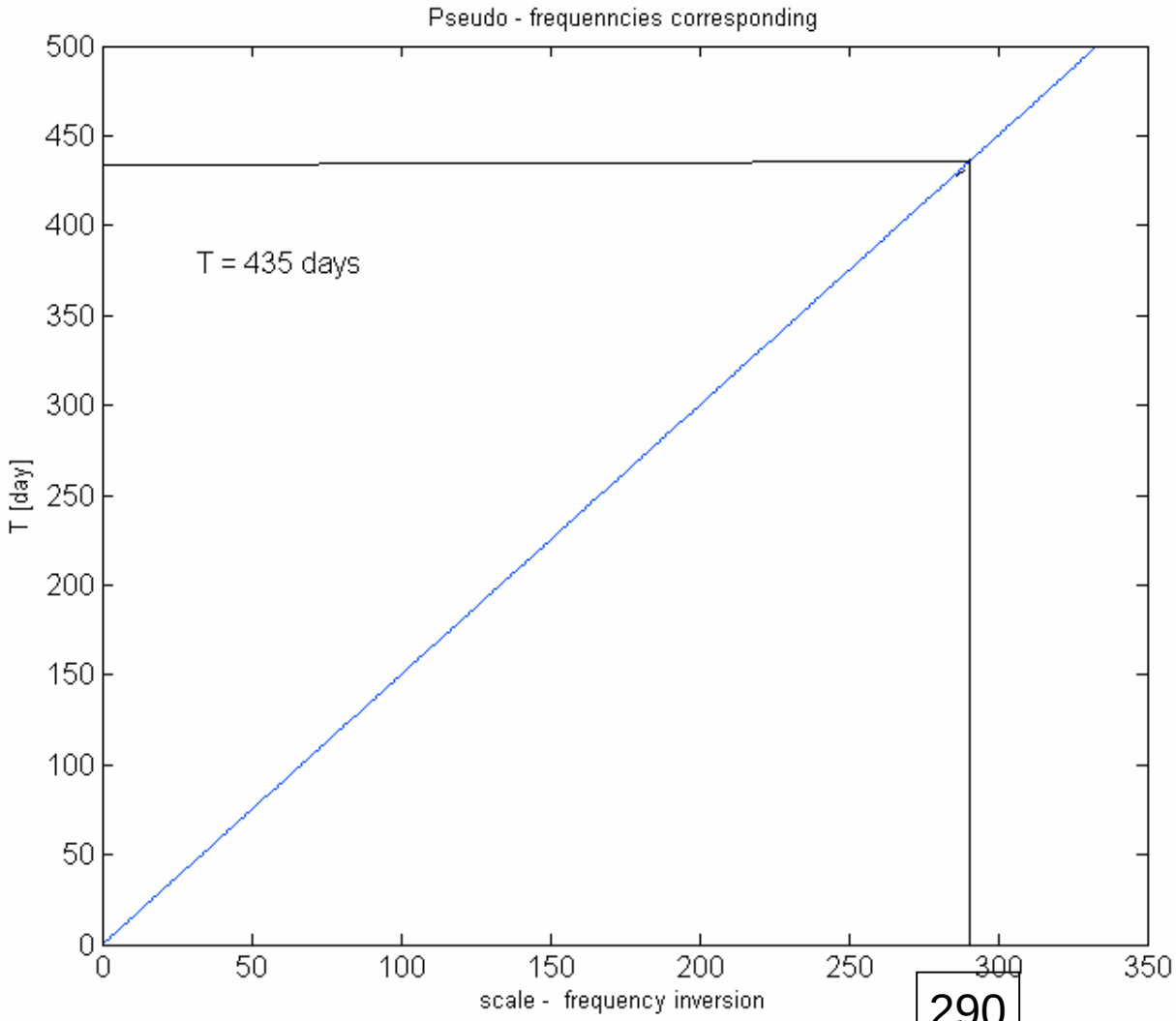
SYMLET 8
N = 8


h GRAZ
0.1
0.09
0.08
GRAZ-h Periodogram
25
20
15
10
5
0
X: 0.002794
Y: 21.5
0
0.005
0.01
0.015
0.02
0.025
cycles/day
FFT
802
642
482
322
162
2
scale - frequency inversion
500
1000
1500
2000
2500
3000
3500
4000
4500
time - day

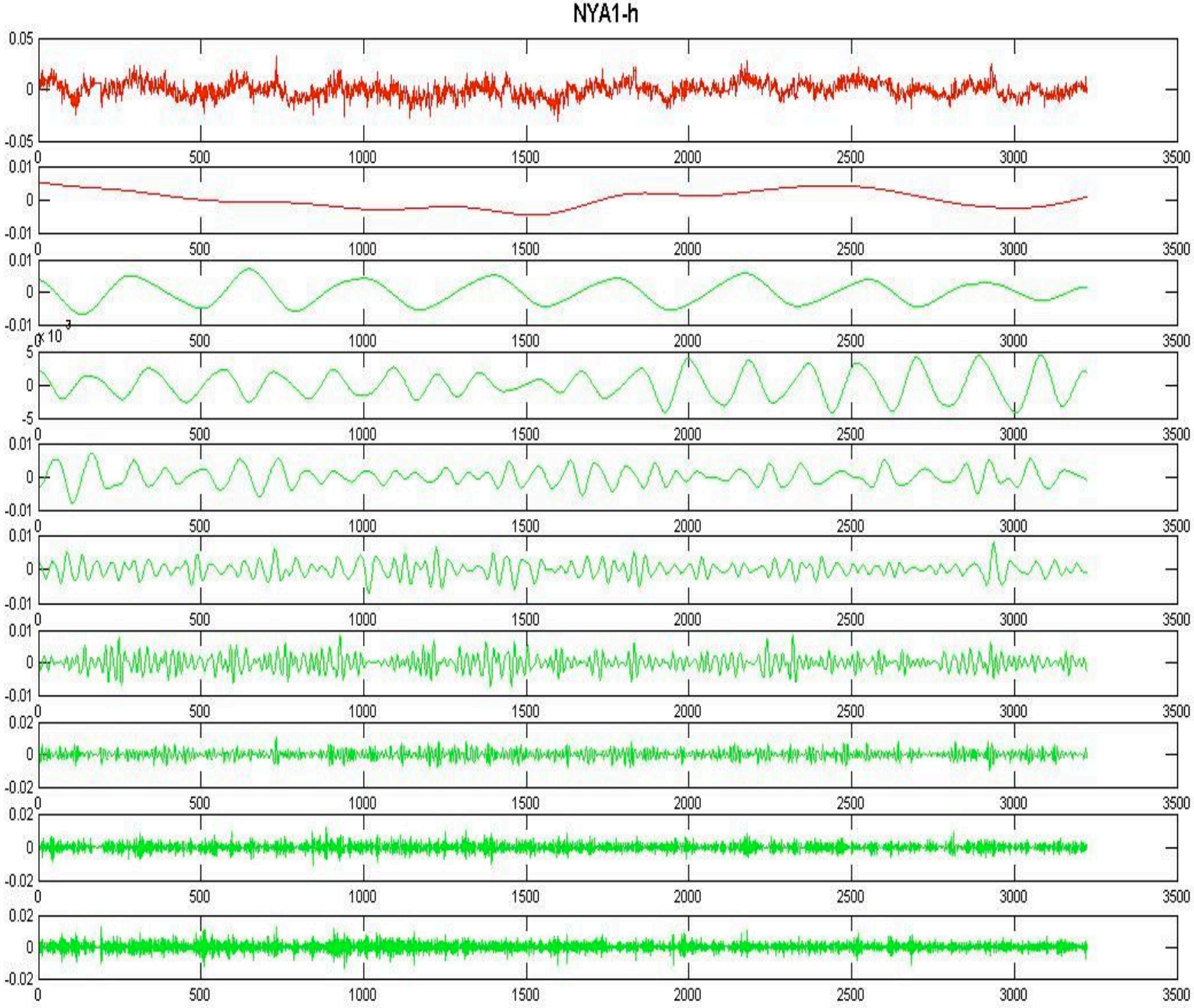

h GRAZ
scale - frequency inversion
3042
2882
2722
2562
2402
2242
2082
1922
1762
1602
1442
1282
1122
962
802
642
482
322
162
2
500
1000
1500
2000
2500
3000
3500
4000
4500
time - day

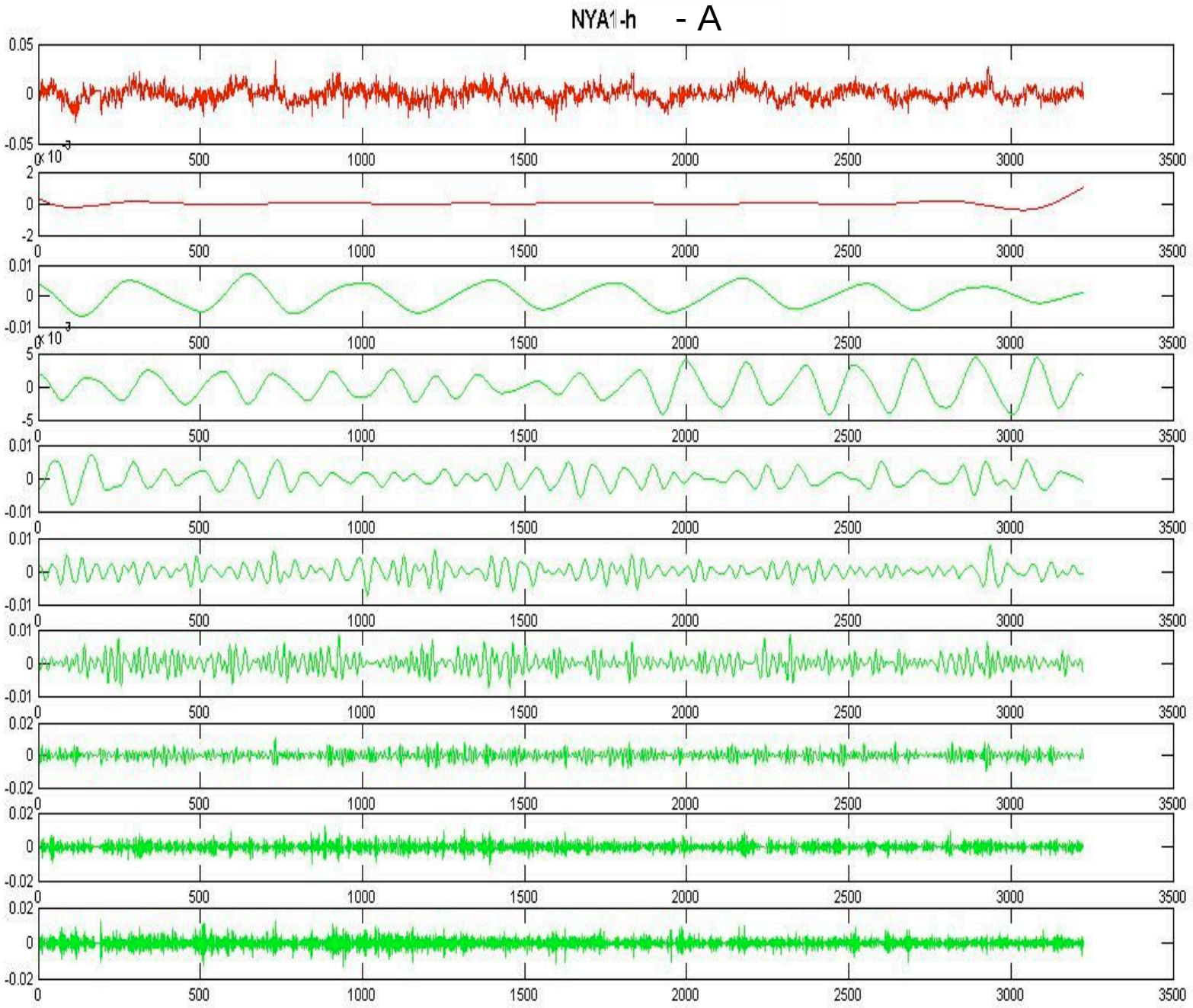

h-A GRAZ
scale - frequency inversion
3042
2882
2722
2562
2402
2242
2082
1922
1762
1602
1442
1282
1122
962
802
642
482
322
162
2
500
1000
1500
2000
2500
3000
3500
4000
4500
time - day

290


Pseudo - frequenncies corresponding
T = 435 days
T [day]
scale - frequency inversion
0
50
100
150
200
250
300
350
400
450
500

290


NYA1-h
0.05
0
-0.05
0.01
0
-0.01
0.01
0
-0.01
x 10^-3
5
0
-5
0.01
0
-0.01
0.01
0
-0.01
0.01
0
-0.01
0.02
0
-0.02
0.02
0
-0.02
0.02
0
-0.02
0
500
1000
1500
2000
2500
3000
3500


NYA1-h - A
0.05
0
-0.05
0
500
1000
1500
2000
2500
3000
3500
x 10^-3
2
0
-2
0.01
-0.01
5
-5
0.02
-0.02

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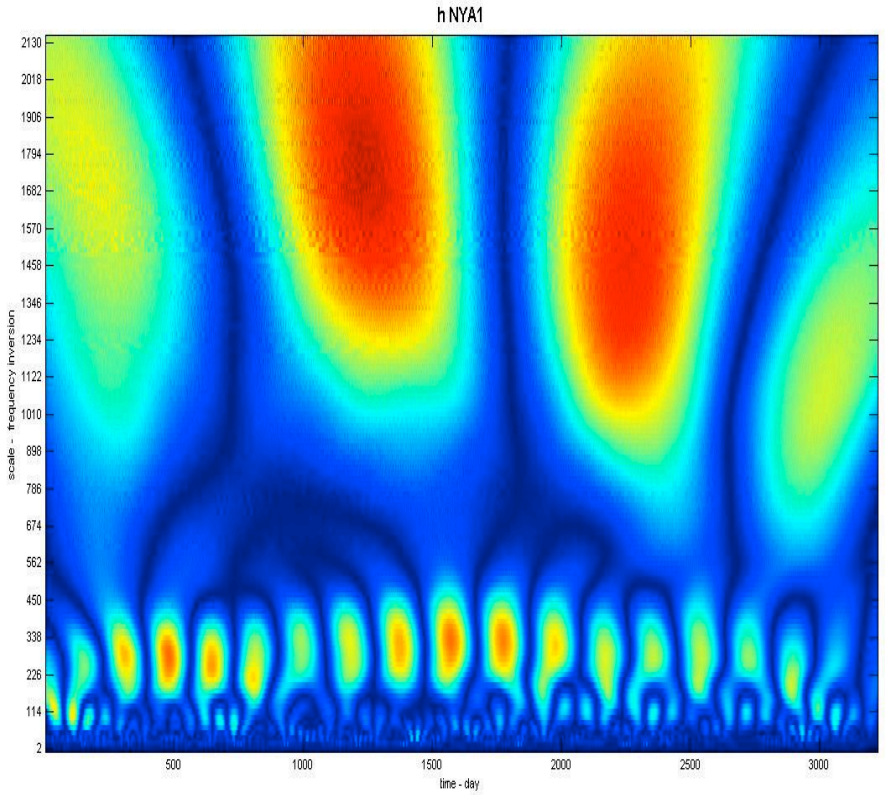

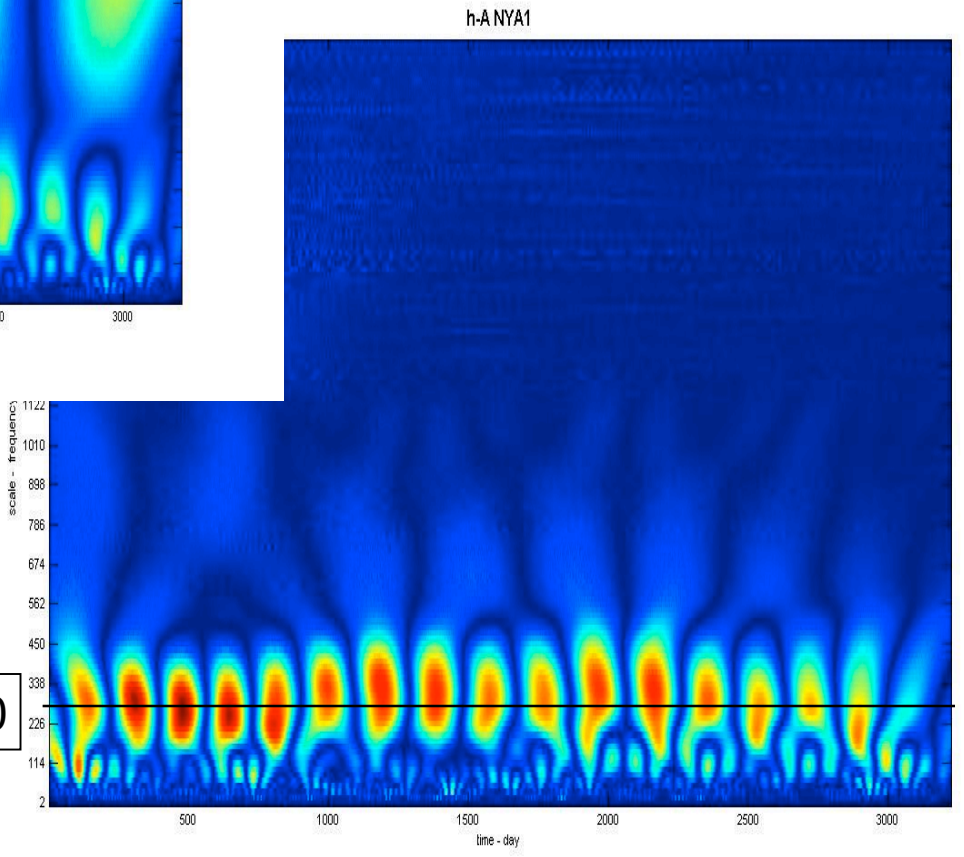

290

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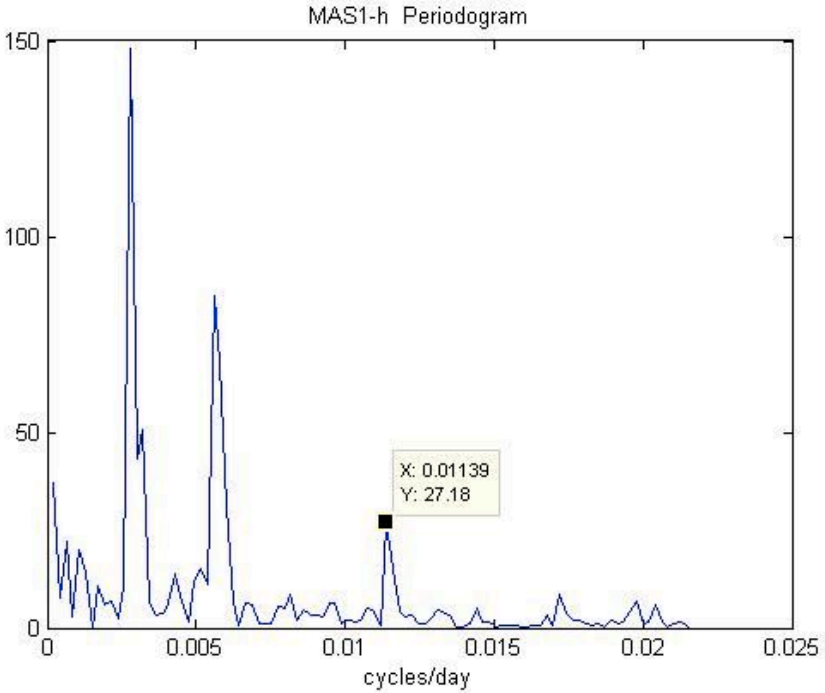
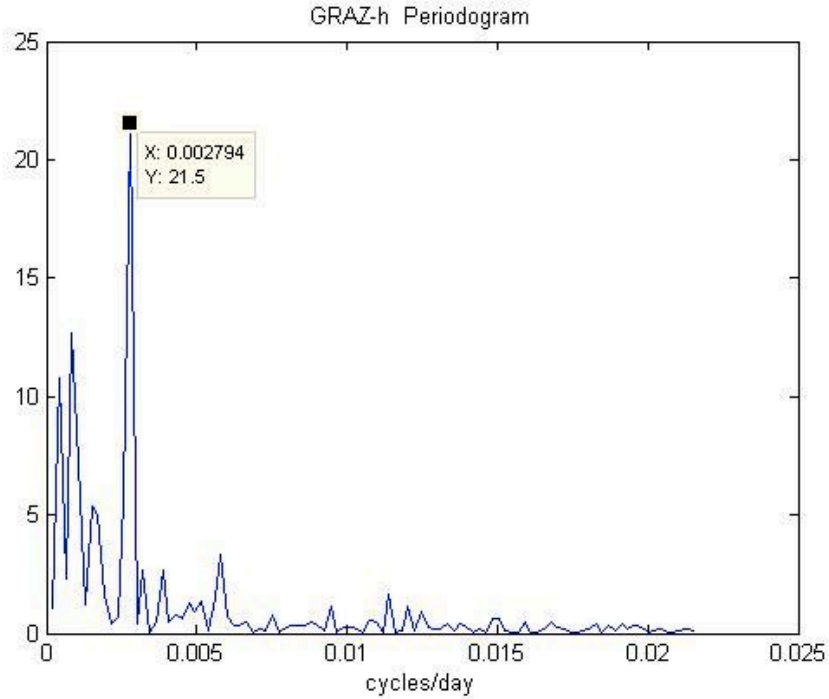
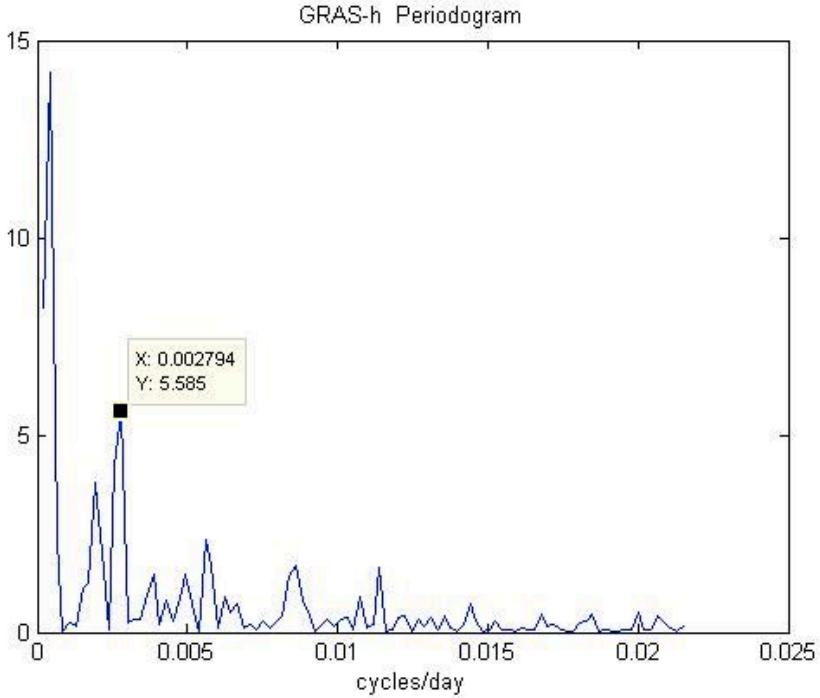
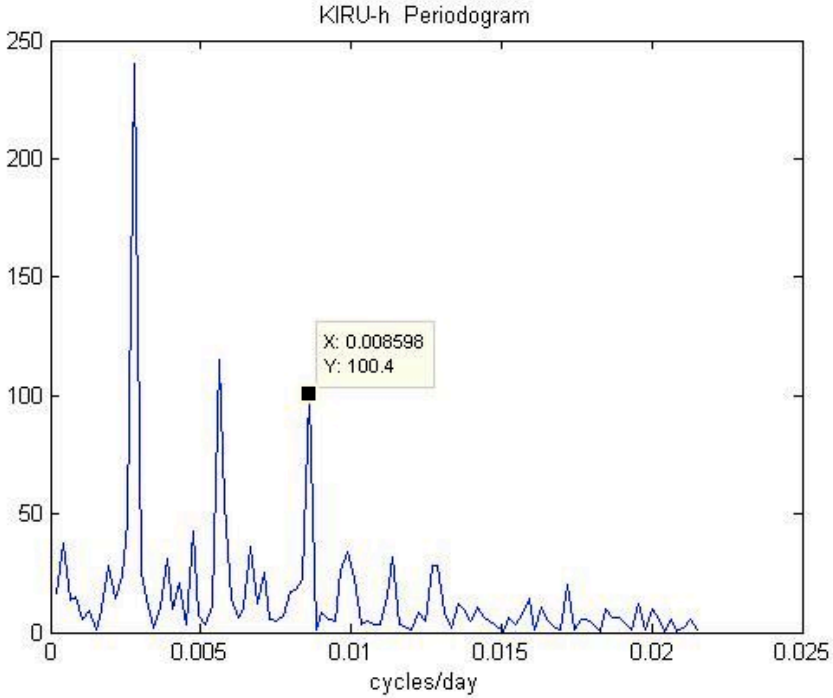

MAS1-h Periodogram
X: 0.01139
Y: 27.18
cycles/day


GRAZ-h Periodogram
X: 0.002794
Y: 21.5
cycles/day

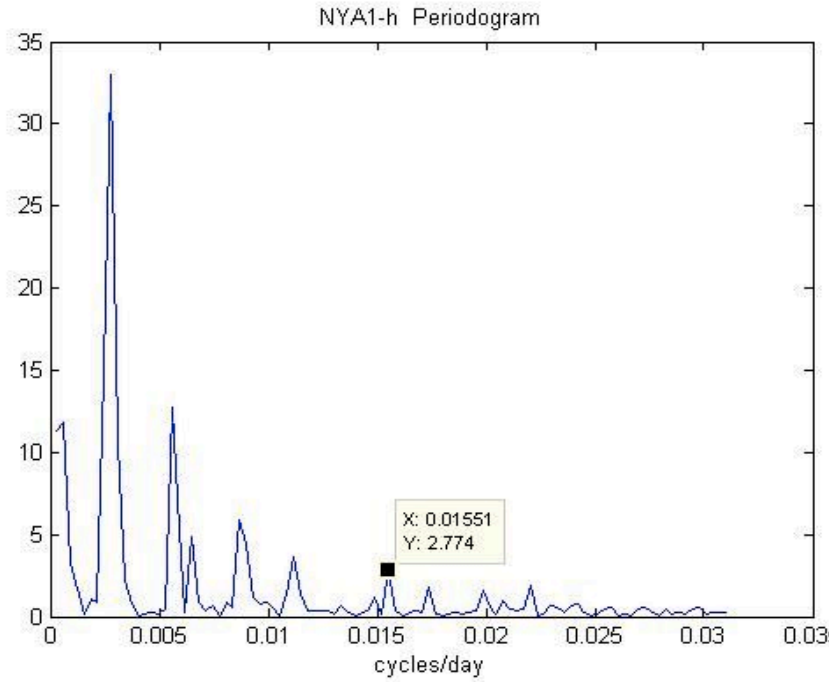
FFT

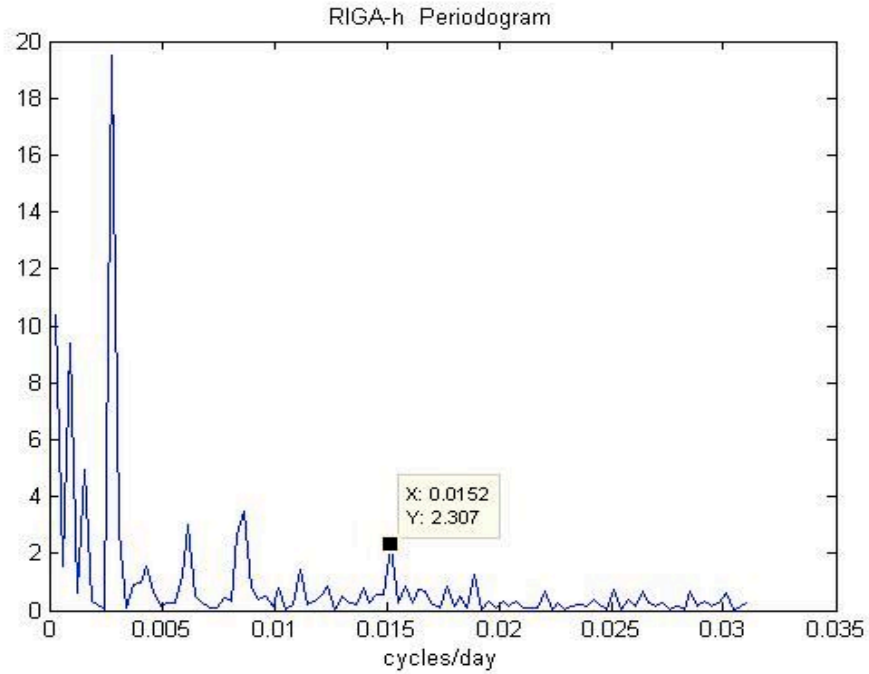

GRAS-h Periodogram
X: 0.002794
Y: 5.585
cycles/day


KIRU-h Periodogram
X: 0.008598
Y: 100.4
cycles/day

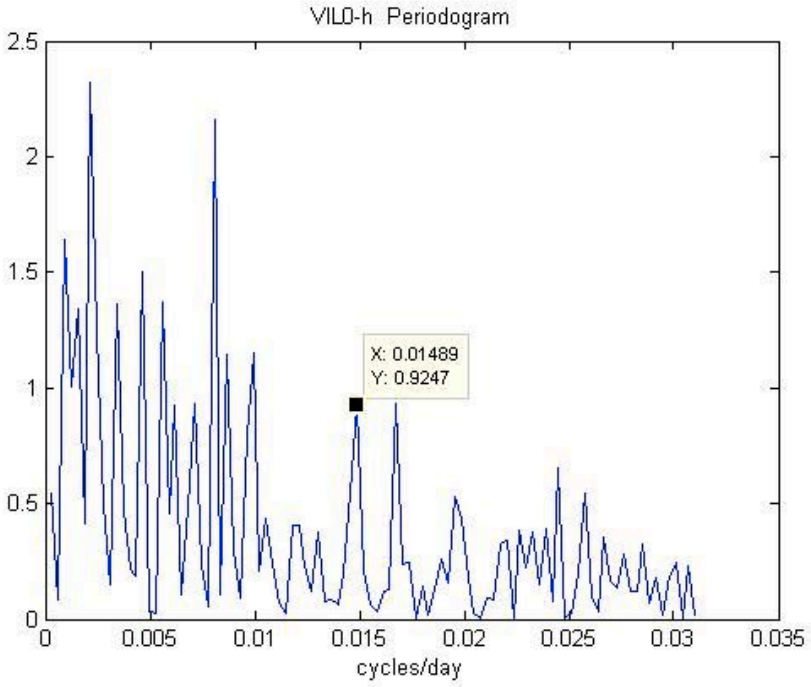
LONG

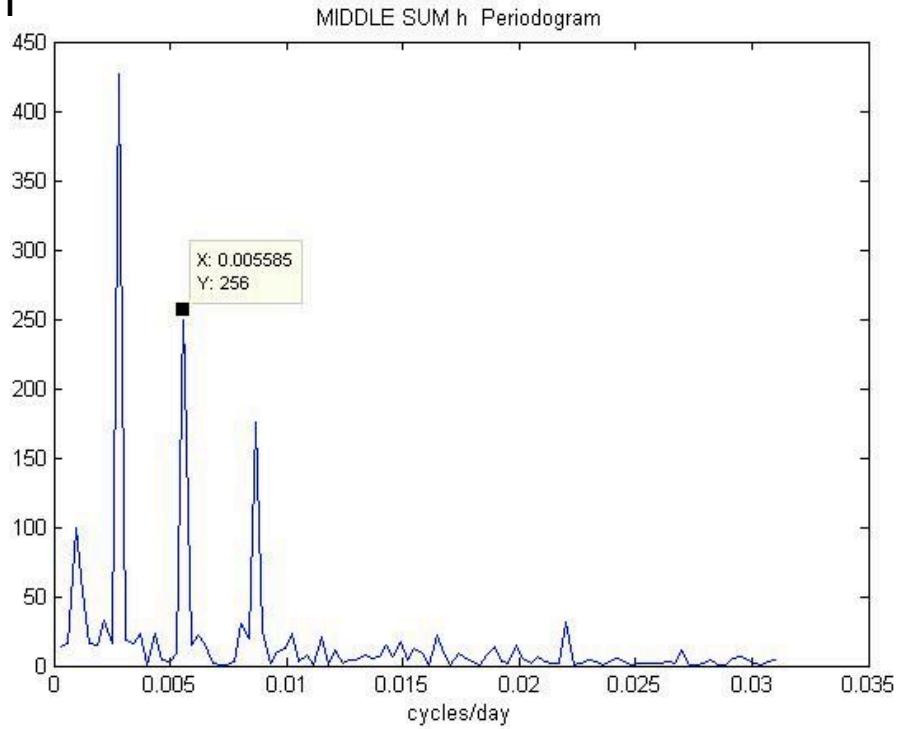
freqency	ZIMM	VILL	POTS	ONSA	METS	MAS1	KOSG	KIRU	JOZE	GRAZ	GRAS
cycles/day	0,000645	0,00086	0,00043	0,000645	0,00172	0,000645	0,00086	0,001935	0,00086	0,00086	0,00043
day	1551	1163	2326	1551	581	1551	1163	517	1163	1163	2326
cycles/day	0,001505	0,001505	0,001505	0,00172	0,002365	0,001075	0,001505	0,002794	0,001505	0,001505	0,001935
day	664	664	664	581	423	930	664	358	664	664	517
cycles/day	0,001935	0,001935	0,00258	0,00215	0,002794	0,002794	0,002365	0,003869	0,001935	0,002794	0,002794
day	517	517	388	465	358	358	423	258	517	358	358
cycles/day	0,002365	0,002365		0,00258	0,003439	0,003224		0,004729	0,002794	0,005804	0,005589
day	423	423		388	291	310		211	358	172	179
cycles/day	0,005589	0,002794		0,003224	0,003869	0,004299		0,005589	0,005904		
day	179	358		310	258	233		179	169		
cycles/day		0,003439		0,003869	0,005804	0,005589		0,006664			
day		291		258	172	179		150			
cycles/day				0,005589	0,008383	0,01139		0,008598			
day				179	119	88		116			

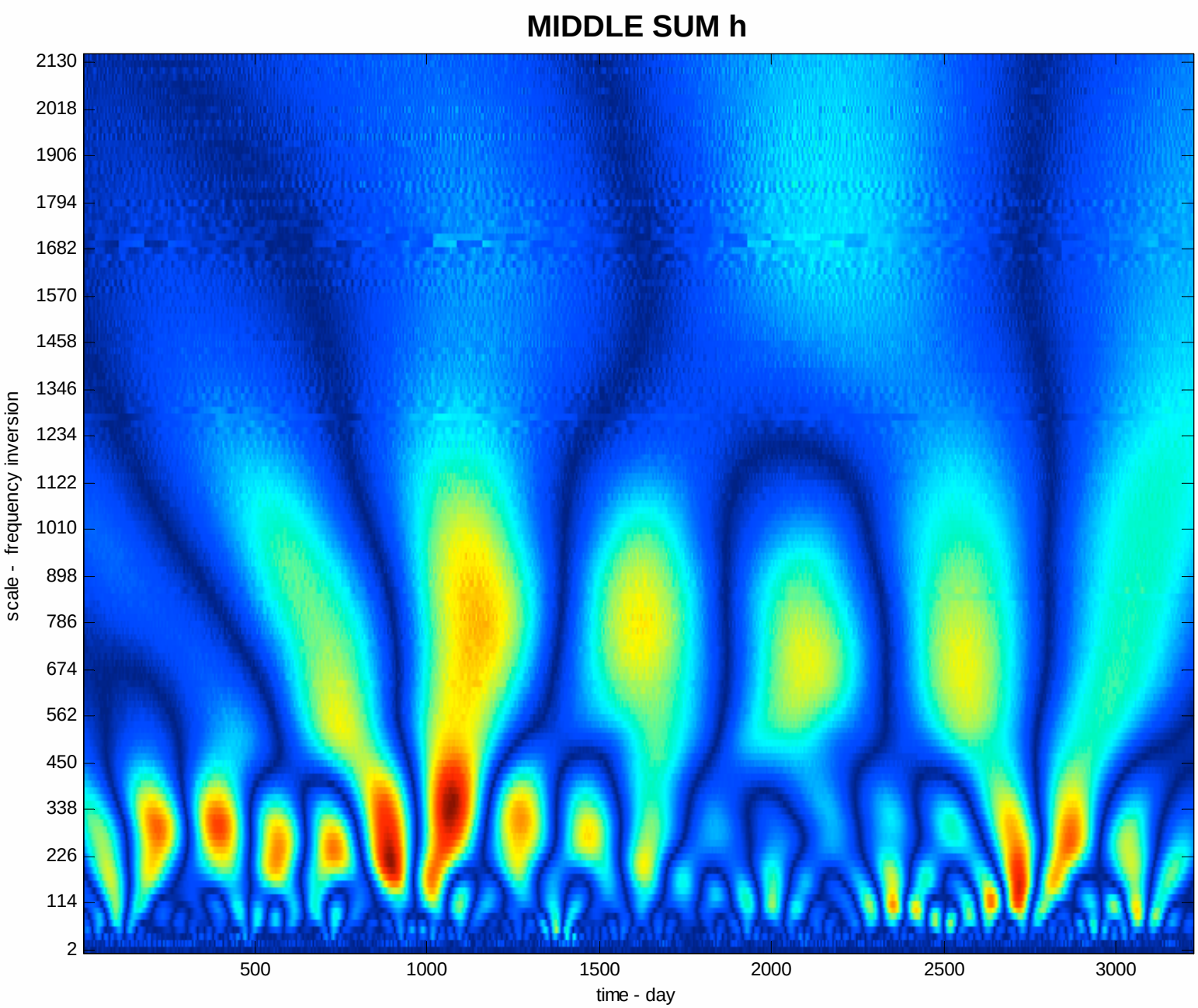

NYA1-h Periodogram
X: 0.01551
Y: 2.774
cycles/day


RIGA-h Periodogram
X: 0.0152
Y: 2.307
cycles/day

FFT

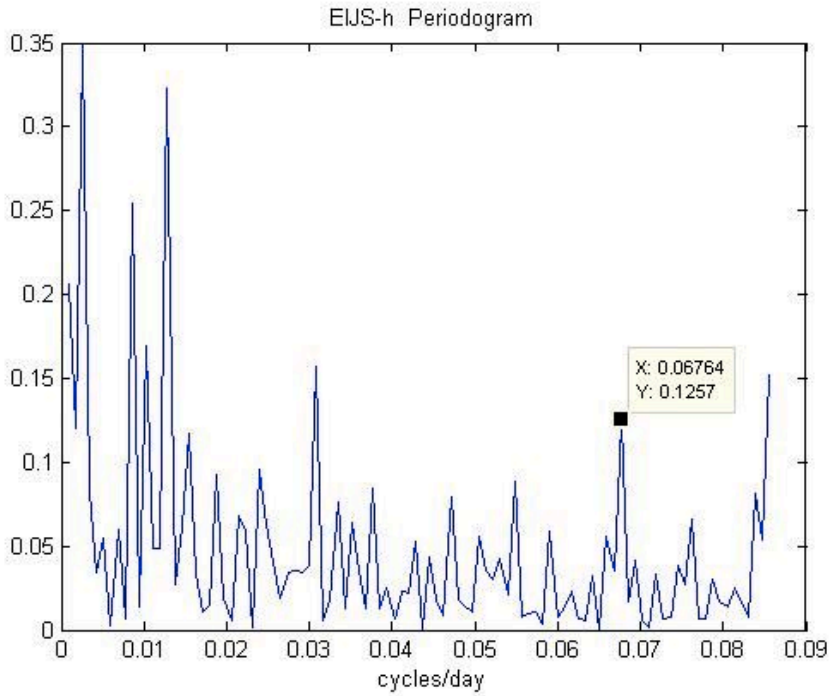

VILO-h Periodogram
X: 0.01489
Y: 0.9247
cycles/day

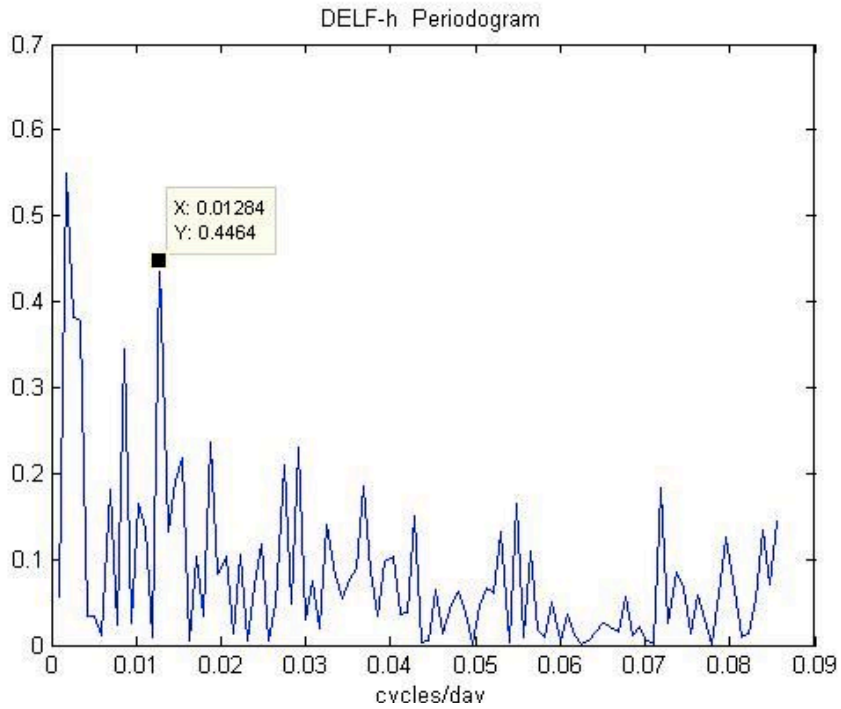

MIDDLE SUM h Periodogram
X: 0.005585
Y: 256
cycles/day


MIDDLE SUM h
scale - frequency inversion
2130
2018
1906
1794
1682
1570
1458
1346
1234
1122
1010
898
786
674
562
450
338
226
114
2
500
1000
1500
2000
2500
3000
time - day

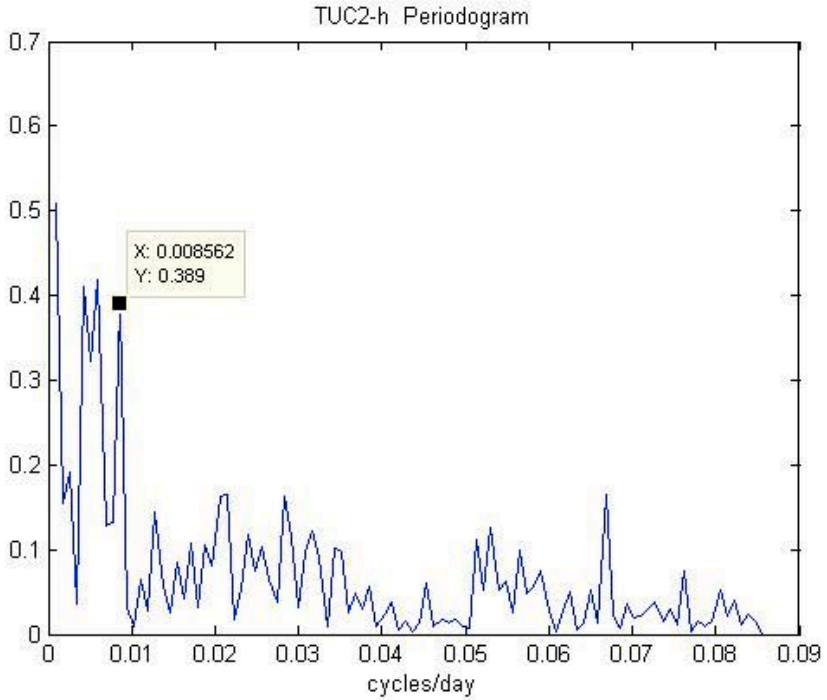
MIDDLE

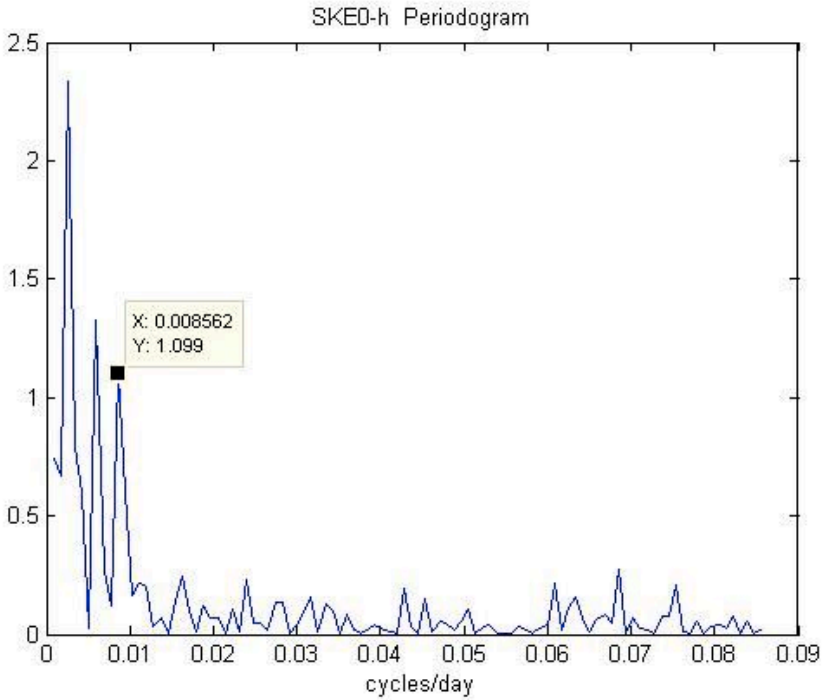
freqency	VIL0	TRO1	RIGA	RAMO	NYA1	MARS	KIR0	JOEN	GLSV	GENO	SUM
ycles/day	0,000931	0,000621	0,000931	0,001241	0,002792	0,001241	0,001241	0,000931	0,002792	0,001551	0,000931
day	1074	1612	1074	806	358	806	806	1074	358	645	1074
ycles/day	0,001551	0,001551	0,001551	0,002792	0,005585	0,002482	0,002172	0,001862	0,005895	0,002482	0,002172
day	645	645	645	358	179	403	460	537	170	403	460
ycles/day	0,002172	0,003413	0,002792	0,004654	0,006516	0,003103	0,002792	0,002792	0,008688	0,008688	0,002792
day	460	293	358	215	153	322	358	358	115	115	358
ycles/day	0,003413	0,005585	0,006205	0,005585	0,008688	0,004344	0,004654	0,005585		0,01148	0,005585
day	293	179	161	179	115	230	215	179		87	179
ycles/day	0,004654	0,03258	0,008688	0,01427	0,01117	0,005895	0,005585	0,006205			0,008688
day	215	31	115	70	90	170	179	161			115
ycles/day	0,005585	0,04716	0,0152		0,01551	0,007136	0,008688	0,008688			0,01644
day	179	21	66		64	140	115	115			61
ycles/day	0,008067					0,008998	0,01644	0,009618			
day	124					111	61	104			
ycles/day	0,008688										
day	115										


EIJS-h Periodogram
X: 0.06764
Y: 0.1257
cycles/day


DELF-h Periodogram
X: 0.01284
Y: 0.4464
cycles/day

FFT


TUC2-h Periodogram
X: 0.008562
Y: 0.389
cycles/day


SKE0-h Periodogram
X: 0.008562
Y: 1.099
cycles/day

SHORT

freqency	SULD	ZYWI	SKE0	DELF	EIJS	TERS	COMO	EVPA	MSEL	TUC2	SUM
cycles/day	0,005993	0,002568	0,002568	0,001712	0,002568	0,002568	0,002568	0,002568	0,002568	0,002568	0,002568
day	167	389	389	584	389	389	389	389	389	389	389
cycles/day	0,01541	0,005137	0,005993	0,006849	0,008562	0,005137	0,008562	0,005993	0,006849	0,004281	0,005993
day	65	195	167	146	117	195	117	167	146	234	167
cycles/day	0,01969	0,007705	0,008562	0,008562	0,01027	0,008562	0,01199	0,008562	0,02055	0,05993	0,008562
day	51	130	117	117	97	117	83	117	49	17	117
cycles/day		0,01027		0,01284	0,01284	0,01541	0,02055	0,01284		0,008562	0,0274
day		97		78	78	65	49	78		117	36
cycles/day		0,0171		0,01884	0,03082	0,01884	0,02483	0,1884		0,0214	0,03253
day		58		53	32	53	40	5		47	31
cycles/day		0,01969			0,0674	0,03767		0,03253		0,02825	0,03767
day		51			15	27		31		35	27
cycles/day		0,02483			0,08562	0,09418				0,06678	
day		40			12	11				15	
cycles/day		0,02997				0,1438				0,1045	
day		33				7				10	

CONCLUSIONS

- Time series of reference stations still can include characteristic oscillation
- Period of 435 days is probably connected with Chandler's effect
- Elimination of 'disturbing' signal frequencies will improve the quality of parametres of reference stations

Thank you for Your attention